

Analysis and Optimization of Ambiguity Function in Radar-Communication Integrated Systems Using MPSK-DSSS

Lan Tang^{ID}, Ke Zhang, Haipeng Dai^{ID}, Pengcheng Zhu^{ID}, and Ying-Chang Liang^{ID}, *Fellow, IEEE*

Abstract—With increasingly sophisticated electromagnetic environment of combat platforms, radar communication integration has attracted wide interests. In this letter, instead of designing a new integrated signal, we employ multiple-phase shift keying direct sequence spread spectrum (MPSK-DSSS) signal as radar communication integrated signal due to its advantage of outstanding correlation. On the one hand, the closed form expression of average ambiguity function is obtained, by which we can analyze the relationship between parameters of integrated signal and radar range resolution. On the other hand, to improve radar peak sidelobe level (PSL) influenced by random communication signals, we propose a joint optimization method based on sequence mapping at the transmitter and sidelobe suppression at the radar receiver. Simulation results show that the sidelobe levels of the optimized integrated signal is effectively suppressed at the cost of slight communication rate loss.

Index Terms—Radar-communication integration, MPSK-DSSS, average ambiguity function, peak sidelobe level.

I. INTRODUCTION

IN MODERN warfare, combat platforms have to be equipped with a large number of electronic devices which may cause serious electromagnetic compatibility problems. Thus, radar and communication integration becomes a focus of research [1] owing to the advantage of reducing interference.

Main integrated approaches can be classified into two categories: signal-multiplexing and signal-sharing. The former generates radar signals and communication signals respectively, and then superimposes them at the transmitter by time division multiplexing [2] or code division multiplexing [3]. The latter only takes a single signal to realize radar detection

and information transmission at the same time [4]–[11]. The integrated signals in [4], [5] are designed by combining linear frequency modulation (LFM) and continuous-phase modulation (CPM). Both [4] and [5] analyzed the performance of an integrated signal based on LFM-MSK (minimum shift keying). Reference [6] proposed a phase-modulation based method by embedding information into the radar pulse and developed three signalling strategies to ensure that the receiver can detect the embedded information.

The signal integration in [7]–[11] is based on spread spectrum (SS) technology. In [8], SS technology is applied to radar-embedded communication to provide low probability of intercept communication. Reference [9] proposed integrated Orthogonal Frequency Division Multiplexing (OFDM) waveform based on SS modulation to increase the peak-to-sidelobe level (PSL). In [10], an integrated radar-communication scheme using time-modulated frequency diverse multiple-input multiple-output array was proposed, where information bits were embedded via spread sequence technique. Reference [11] verified by simulation results that low sidelobe level can be achieved by selecting sufficient long code sequence from a proper SS code family. This letter above did not investigate the effect of random SS signals on radar performance and develop tradeoff methods considering both communication and radar performance.

In this letter, instead of designing a new integrated signal, we adopt an existing signal based on M -ary phase shift keying (MPSK) and direct sequence spread spectrum (DSSS) as the integrated signal owing to its splendid correlation performance [12]. The main contribution of this letter is two-fold: 1) Considering ambiguity function is an important tool to analyze radar waveform, we analyze the performance of average ambiguity function utilizing correlation properties of M-sequence [13], and draw the relationship between parameters of MPSK-DSSS signals and radar range resolution. It provides a theoretical basis for our rational choice of integrated signal parameters. 2) We propose a joint optimization method based on sequence mapping at the transmitter and sidelobe suppression at the radar receiver to reduce the impact of randomness of MPSK-DSSS signals on PSL.

II. INTEGRATED SYSTEM MODEL BASED ON MPSK-DSSS

As shown in Fig. 1, we consider a dual-function radar-communication system which includes one integrated transmitter, one radar receiver and one communication receiver. The transmitter transmits pulses carrying information periodically to perform the data transmission and radar sensing simultaneously. Considering the good autocorrelation property of SS signals, DSSS signals are adopted as pulse waveform. In each pulse period T_p , N spread spectrum MPSK signals

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L. Tang and K. Zhang are with the School of Electronic Science and Engineering, Nanjing University, Nanjing 210093, China (e-mail: tanglan@nju.edu.cn; 913634986@qq.com).

H. Dai is with the State Key Laboratory for Novel Software Technology, Nanjing University, Nanjing 210023, China, and also with the Key Laboratory of Broadband Wireless Communication and Sensor Network Technology, Ministry of Education, Nanjing University of Posts and Telecommunications, Nanjing, China (e-mail: haipengdai@nju.edu.cn).

P. Zhu is with the National Mobile Communications Research Laboratory, Southeast University, Nanjing 210096, China (e-mail: p.zhu@seu.edu.cn).

Y.-C. Liang is with the Center for Intelligent Networking and Communications, University of Electronic Science and Technology of China, Chengdu 611731, China (e-mail: liangyc@ieee.org).

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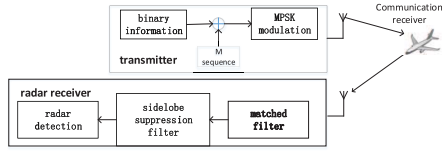


Fig. 1. Radar-communication integrated system.

are transmitted. The complex envelop of transmitted signals is

$$s(t) = \sum_{n=0}^{N-1} \sum_{l=0}^{L-1} e^{j(\phi_n + b_l \Delta \phi_n)} g(t - (nL + l)T_c) = \sum_{q=0}^{NL-1} d_q g(t - qT_c) \quad (1)$$

where L is spreading factor of M sequence, $b_l \in \{0, 1\}$ is the l th bit of an M sequence, T_c is chip period, $g(\cdot)$ is the rectangular pulse with amplitude A and duration T_c , $d_q = e^{j\phi_q}$, ϕ_n and $\Delta \phi_n$ are the phase and chip phase difference of the n th MPSK symbol, respectively. When $\phi_n = \phi^m$ ($1 \leq m \leq M$), the chip phase difference $\Delta \phi_n$ may be $\Delta \phi^M$ or $2\pi - \Delta \phi^M$. It can be observed from Gray mapping that $\Delta \phi^M = \{\pi, \pi, \frac{5\pi}{4}, \frac{5\pi}{4}, \frac{21\pi}{16}, \frac{21\pi}{16}, \frac{85\pi}{64}, \frac{85\pi}{64}\}$ when $M = 2^k$ ($k = 1, \dots, 8$). The mapping method from bits to chip phases will effect the communication and radar performance, as discussed in the next section.

At the communication receiver, the received equivalent low-pass signal in the l th chip of the n th symbol is

$$r(t) = \alpha s(t) + z(t), \quad (nL + l)T_c \leq t \leq (nL + l + 1)T_c \quad (2)$$

where α is the channel coefficient between the transmitter and communication receiver, and $z(t)$ is AWGN with power spectral density n_0 . To demodulate the n th symbol, $r(t)$ is passed through a matched filter with impulse response $g(t - T_c)$, and the sampled output is multiplied by $e^{jb_l \Delta \phi^M}$ or $e^{-jb_l \Delta \phi^M}$ to remove the effect of spread sequence.

At the radar receiver, target echoes are passed through a matched filter with impulse response $s^*(T_p - t)$, followed by a sidelobe suppression filter to reduce the impact of random signals on radar detection.

III. ANALYSIS OF COMMUNICATION PERFORMANCE AND AVERAGE AMBIGUITY FUNCTION

A. SER Performance

We extend the error probability analysis of BPSK-DSSS in [14] to the case of MPSK-DSSS. Suppose the phase and chip phase difference of the n th symbol is ϕ^m and $\Delta \phi^M$, respectively. The sampled output from matched filter is $y_{nl} = \alpha A^2 T_c e^{j\phi_n l} + \tilde{z}_{nl}$ where $\tilde{z}_{nl} = \int_0^{T_c} g(t) z(t + (nL + l)T_c) dt$. In the demodulator, the correlation metrics $CM_1 = \sum_{l=0}^{L-1} e^{-jb_l \Delta \phi^M} y_{nl}$ and $CM_2 = \sum_{l=0}^{L-1} e^{jb_l \Delta \phi^M} y_{nl}$ are computed and compared. we can derive $CM_1 \sim \mathcal{CN}(\alpha LA^2 T_c e^{j\phi^m}, \sigma^2)$ and $CM_2 \sim \mathcal{CN}(\frac{\alpha LA^2 T_c}{2} e^{j\phi^m} (1 + e^{j2\Delta \phi^M}), \sigma^2)$, where $\sigma^2 = LA^2 T_c n_0$. When $|CM_1| > |CM_2|$, the decision error probability based on CM_1 is probability of error for MPSK $P_{MPSK}(\gamma_s) \approx 2Q(\sqrt{\gamma_s} \sin(\pi/M))$ where $\gamma_s = \frac{\alpha^2 LA^2 T_c}{n_0}$ [14]. Hence, the symbol error probability for MPSK-DSSS is

$$P_s = 1 - \Pr(|CM_1| > |CM_2|)(1 - P_{MPSK}(\gamma_s)). \quad (3)$$

We can calculate $\Pr(|CM_1| > |CM_2|)$ via numerical integration since (CM_1, CM_2) obey complex Gaussian distribution.

Simulation results show that when $\gamma_s > 15\text{dB}$, $\Pr(|CM_1| > |CM_2|) \approx 1$. Therefore, in the normal SNR regime, $P_s \approx P_{MPSK}$. Specially, when $M = 2$ or 4 , $\Delta \phi^M = \pi$ and $CM_1 = CM_2 \sim \mathcal{CN}(\alpha LA^2 T_c e^{j\phi^m}, \sigma^2)$. In this case, $P_s = P_{MPSK}(\gamma_s)$.

B. Average Ambiguity Function

Ambiguity function is an important tool to analyze and design radar waveform. Unlike conventional pulse compression waveforms, in radar-communication integrated systems, $s(t)$ is a stochastic waveform signal. Hence, the ambiguity function of $s(t)$ is also a random function. To remove the influence of random communication signals, we will analyze average ambiguity function of $s(t)$. From the analytical result, we can obtain some insights about the effect of communication parameters on the radar performance.

The ambiguity function of the MPSK-DSSS signal is

$$\begin{aligned} \chi(\tau, f_d) &= \int_{-\infty}^{+\infty} s(t) s^*(t - \tau) e^{j2\pi f_d t} dt \\ &= \sum_{q=0}^{NL-1} \sum_{q'=0}^{NL-1} d_q d_{q'}^* e^{j2\pi q T_c f_d} \chi_g(\tau - (q - q')T_c, f_d), \end{aligned} \quad (4)$$

where τ and f_d denote time delay and doppler shift, $\chi_g(\tau, f_d)$ is the ambiguity function of $g(t)$. When $|\tau| \leq T_c$, $\chi_g(\tau, f_d) = A^2(T_c - |\tau|) e^{j\pi f_d (T_c + \tau)} \sin c(f_d(T_c - |\tau|))$. Otherwise, $\chi_g(\tau, f_d) = 0$. Hence, when τ is in different intervals, the ambiguity function can be rewritten as the following fractional function

$$\chi(\tau, f_d) = \begin{cases} \sum_{q=0}^{NL-1-\tilde{l}} d_{q+\tilde{l}} d_q^* e^{\gamma(q+\tilde{l})} \chi_g(|\tau| - \tilde{l}T_c, f_d) \\ + \sum_{q=0}^{NL-2-\tilde{l}} d_{q+\tilde{l}+1} d_q^* e^{\gamma(q+\tilde{l}+1)} \chi_g(|\tau| - (\tilde{l}+1)T_c, f_d), \\ \quad \tilde{l}T_c \leq |\tau| \leq (\tilde{l}+1)T_c, 0 \leq \tilde{l} \leq NL-2 \\ d_{NL-1} d_0^* e^{\gamma(NL-1)} \chi_g(|\tau| - (NL-1)T_c, f_d), \\ \quad (NL-1)T_c \leq |\tau| \leq NL T_c \\ 0, \quad \text{otherwise} \end{cases} \quad (5)$$

where $\gamma = j2\pi T_c f_d$ and $\tilde{l}$ represents the chip delay. (5) holds because only terms that satisfy $|\tau - \tilde{l}T_c| \leq T_c$ have nonzero value and other terms are vanishing. Specifically, when $|\tau| > NL T_c$, $\chi(\tau, f_d) = 0$ and when $|\tau| \leq NL T_c$, $\chi(\tau, f_d)$ is a random function related to the product of random signals.

Therefore, to obtain $\mathbb{E}(\chi(\tau, f_d))$, we first calculate $\mathbb{E}(d_{q+\tilde{l}} d_q^*)$. Since $E(d_{q+\tilde{l}} d_q^*)$ depends on $\tilde{l}$, we consider the following three cases: 1) $\tilde{l} = 0$. In this case, $\mathbb{E}(d_{q+\tilde{l}} d_q^*) = 1$. 2) $L < |\tilde{l}| \leq NL - 1$. In this case, d_q and $d_{q+\tilde{l}}$ belong to different MPSK symbols so that they are uncorrelated. Assuming MPSK symbols have equal probability of occurrence, we have $\mathbb{E}(d_q) = \frac{1}{M} \sum_{m=0}^{M-1} e^{j\frac{2\pi}{M}m} = 0$. As a result, $\mathbb{E}(d_{q+\tilde{l}} d_q^*) = 0$. 3) $0 < |\tilde{l}| \leq L$. In this case, d_q and $d_{q+\tilde{l}}$ may be in different MPSK symbols or a same MPSK symbol. When d_q and $d_{q+\tilde{l}}$ belong to a same MPSK symbol, we calculate $\mathbb{E}(d_{q+\tilde{l}} d_q^*)$ by utilizing the correlation property and balance property of M sequence and get the following result,

Lemma: In an MPSK-DSSS symbol,

$$\mathbb{E}(d_{q+\tilde{l}} d_q^*) \approx \frac{1}{L} \left(\frac{L-1}{2} + \frac{L+1}{2} \cos \Delta \phi^M \right). \quad (6)$$

Proof: See the Appendix. ■

Based on the result in (5) and the above Lemma, $E(\chi(\tau, f_d))$ can be calculated approximately as

$$E(\chi(\tau, f_d)) = \begin{cases} \frac{1-e^{NL\gamma}}{1-e^\gamma} \left[\chi_g(|\tau|, f_d) + \frac{e^\gamma(1-e^{(L-1)\gamma})}{1-e^{L\gamma}} \chi_g(|\tau|-T_c, f_d)\beta \right], & |\tau| \leq T_c \\ \frac{\beta e^{\tilde{l}\gamma}(1-e^{NL\gamma})}{(1-e^\gamma)(1-e^{L\gamma})} \left[(1-e^{(L-1)\gamma}) \chi_g(|\tau|-\tilde{l}T_c, f_d) \right. \\ \left. + e^\gamma(1-e^{(L-\tilde{l}-1)\gamma}) \chi_g(|\tau|-(\tilde{l}+1)T_c, f_d) \right], & T_c < |\tau| \leq (\tilde{l}+1)T_c, 1 \leq \tilde{l} \leq (L-2) \\ \frac{\beta e^{(L-1)\gamma}(1-e^{NL\gamma})}{(1-e^\gamma)} \chi_g(|\tau|-(L-1)T_c, f_d), & (L-1)T_c < |\tau| \leq LT_c \\ 0, & \text{otherwise} \end{cases} \quad (7)$$

where $\beta = \frac{1}{L}(\frac{L-1}{2} + \frac{L+1}{2} \cos \Delta\phi^M)$. When $f_d = 0$, (7) can be simplified as

$$E(\chi(\tau, 0)) = \begin{cases} A^2([N(L-1)\beta - NL]|\tau| + NL T_c), & |\tau| \leq T_c \\ -A^2 N\beta(|\tau| - LT_c), & T_c < |\tau| \leq LT_c \\ 0, & |\tau| > LT_c. \end{cases} \quad (8)$$

Delay resolution constant, which reflects radar resolution characteristics, is defined as $A_\tau = \frac{\int_{-\infty}^{\infty} |\mathbb{E}(\chi(\tau, 0))|^2}{|\chi(0, 0)|^2}$. With the increase of A_τ , resolution performance of distance ambiguity function will deteriorate. By substituting (8) into A_τ , A_τ can be calculated as

$$A_\tau = \frac{2}{3} T_c \left[\frac{(L-1)^2}{L} \beta^2 + \frac{L-1}{L} \beta + 1 \right]. \quad (9)$$

It can be seen that A_τ is a linear function of T_c . The relationships among A_τ , M and L are shown in Section V.

IV. OPTIMIZATION OF INSTANTANEOUS AMBIGUITY FUNCTION PERFORMANCE

Influenced by random communication signals, the ambiguity function of some transmit waveform $s(t)$ has many sidelobes which interfere with accurate detection of weak targets. Moreover, compared with 13.2dB PSL of classical LFM signal, instantaneous PSL of some MPSK-DSSS waveform is low. To suppress sidelobes, the output of matched filter is passed through a recursive least square (RLS) filter. For a given $s(t)$, the impulse response w_s of the filter is optimized to minimize $\|\chi_s \otimes w_s - d\|^2$ with RLS algorithm [15] ($w_s(0) = \mathbf{0}, \delta = 10^{-3}, \lambda = 1$), where χ_s is sampled ambiguity function of $s(t)$, d is Dirac Delta function and $\otimes$ denotes convolution. We set $\mathcal{L}(w_s) = \mathcal{L}(\chi_s)$ where $\mathcal{L}(\cdot)$ denotes the length of vector.

For the transmit waveform $s(t)$ including N spread spectrum MPSK symbols, M^N possible waveforms can be generated. If all waveforms are used to transmit information, we will find that for some waveforms, PSL of output of the RLS filter cannot meet the requirement of radar detection performance (blue line in Fig. 3(a)). Therefore, tradeoff should be achieved by selecting approximate waveforms with high PSL from M^N possible transmit waveforms. Let $\mathcal{U}_M$ denote the set of selected waveforms and $|\mathcal{U}_M|$ be the size of $\mathcal{U}_M$. At the transmitter, in each pulse period, $\log |\mathcal{U}_M|$ bits are added some redundant bits and mapped to the waveform in $\mathcal{U}_M$. At the communication receiver, the received signals are symbol by symbol demodulated (as stated in Section III-A), and

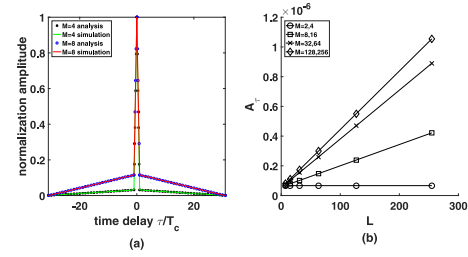


Fig. 2. (a) Analytical and simulation results for average ambiguity function; (b) Relationship between A_τ and parameters.

then information bits are recovered according to mapping relationship between waveform and information bits. We assume that when SER is smaller than a small value ε , $\log |\mathcal{U}_M|$ bits can be correctly received. The rate maximization problem with communication and radar performance constraints is expressed as

$$\begin{aligned} \max_{\mathcal{U}_M, M} \quad & \log |\mathcal{U}_M| / N \\ \text{s.t.} \quad & P_s \leq \varepsilon, \text{PSL}(\chi_s \otimes w_s) > 13.2\text{dB}, \quad \forall s(t) \in \mathcal{U}_M. \end{aligned} \quad (10)$$

For a given γ_s , M is determined by selecting the highest modulation order which satisfies $P_s \leq \varepsilon$. Then, the optimal $\mathcal{U}_M$ is obtained by exhaustively searching waveforms which satisfy the second constraint in (10). Specifically, for each $s(t)$, w_s^* is obtained by minimizing $\|\chi_s \otimes w_s - d\|^2$. If PSL of $\chi_s \otimes w_s^*$ is larger than 13.2dB, $s(t)$ is added into $\mathcal{U}_M$. When $\mathcal{L}(\chi_s) = 2NL$, the computation complexity of RLS is $\mathcal{O}(16N^3L^3)$. Hence, the total computation complexity is $\mathcal{O}(16M^N N^3 L^3)$.

Finally, we stress that sequences in $\mathcal{U}_M$ can be obtained and stored offline so that the sequence mapping can be implemented in real time.

V. SIMULATION RESULTS

In this section, some simulation results are presented to validate theoretical analysis results and effectiveness of the proposed method. In the following simulations, if there is no special explanation, $L = 31$ and $T_c = 10^{-7}$ s.

Fig. 2(a) compares analytical results in (8) and simulation results for average ambiguity function by setting $N = \frac{12}{\log M}$. The comparison shows that analytical results of $E(\chi(\tau, 0))$ match simulation results obtained by averaging 10^4 instances yielded by the Monte Carlo method very well. Fig. 2 (b) demonstrates the effects of M and L on A_τ . It can be seen that when T_c is fixed, A_τ will increase with the increasing of M and L . Moreover, L has a greater impact on A_τ with the increasing of M . Therefore, we should choose the smaller value of M , L and T_c to improve radar resolution.

In Fig. 3(a), we set $M = 4$, $N = 3$. Red line is the outputs with the lowest PSL when $s(t)$ is from $\mathcal{U}_M$. Obviously, by selecting appropriate waveforms, sidelobe levels of output of the RLS filter can be suppressed effectively. Meanwhile, the achievable rates $\frac{\log |\mathcal{U}_M|}{N}$ in (10) are given in Fig. 3(b) with $T_c = 3.23 \times 10^{-8}$ s and $T_p = 3 \times 10^{-6}$. It can be observed that rates in Fig. 3(b) are close to $\log M$ without sequence mapping in different SNR regions. Fig. 3 shows that we can satisfy the requirement of radar performance with slight communication rate loss by joint sequence mapping and RLS filtering.

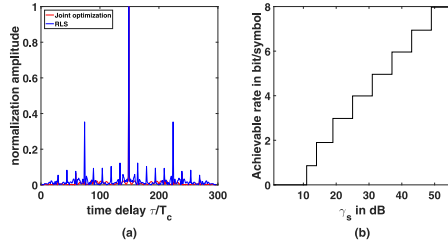


Fig. 3. (a) The outputs of RLS filter with and without sequence mapping; (b) Achievable transmission rate vs. SNR ($\epsilon = 10^{-3}$).

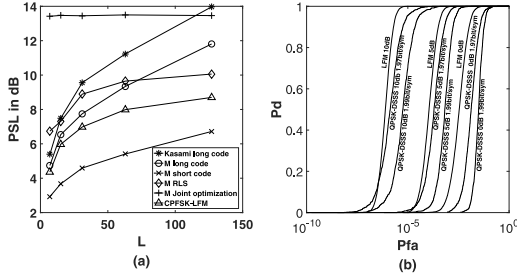


Fig. 4. (a) Comparison of PSL of different SS schemes ($M = 4$, $N = 5$); (b) False alarm probability vs. detection probability.

In Fig. 4(a), we consider a random waveform signal with 4^5 possible waveforms. The lowest PSLs of the random signal are plotted when different encoding principles in [11] are adopted. In addition, PSLs of integrated waveform based on CPFSK-LFM in [4] also are shown for comparison. The bandwidth and period of CPFSK-LFM signal are set to $\frac{1}{T_c}$ and NLT_c . In the proposed method, $\mathcal{U}_M$ and the achievable rate with different L are obtained by solving (10). The achievable rate satisfying PSL requirement is 1.99bit/symbol. It can be observed that by combining sequence mapping and RLS in short-code encoding, higher PSL can be achieved compared with long-code encoding in [11] and CPFSK-LFM, especially when L is small. Fig. 4(b) gives the receiver operating characteristic (ROC) curves of LFM and the proposed method with different received SNR. The probability of false alarm of the proposed method is higher compared to LFM. This behavior occurs primarily because the RLS filtering, in suppressing sidelobes, enhances the noise. Since the impulse response of RLS depends on the ambiguity function of $s(t)$, we can furthermore choose $s(t)$ with low PFA from $\mathcal{U}_M$ as the transmit waveform. As shown in Fig. 4(b), the detection performance of QPSK-DSSS signal has a significant improvement when the transmission rate decreases from 1.99bit/sym. to 1.97bit/sym.

VI. CONCLUSION

In this letter, instead of designing a new integrated signal, we employ an existing MPSK-DSSS signal as the integrated signal. We first derive the closed form expressions of SER, average ambiguity function and delay resolution constant of MPSK-DSSS signal. Then, we propose joint optimization based on RLS filtering at radar receiver and sequence mapping at transmitter to reduce PSL of random MPSK-DSSS signals. Simulation results reveal the effect of communication signal parameters on radar performance, and the tradeoff between communication rate and radar performance.

APPENDIX

Based on ergodicity of stationary stochastic signals, the autocorrelation function of $\{d_q\}$ is calculated approximately: $E(d_{q+\tilde{l}}d_q^*) \approx \frac{1}{L} \sum_{q=0}^{L-1} d_q d_{(q+\tilde{l}) \bmod (L-1)}^*$ where $\tilde{l}$ is the chip delay. Each term in above equation depends on the chip phase difference. When $M \leq 256$, the chip phase difference has three possible values: 0, $\Delta\phi^M$ and $2\pi - \Delta\phi^M$.

We suppose that $\{d_q\}_{q=0}^{L-1}$ is the chip sequence generated by a given M-sequence M_0 in an MPSK symbol. By rotating d_q $\tilde{l}$ times, we can get $\{d_{(q+\tilde{l}) \bmod (L-1)}\}_{q=0}^{L-1}$, which is the chip sequence generated by another M-sequence $M_{\tilde{l}}$. $x_q = d_q d_{(q+\tilde{l}) \bmod (L-1)}^*$ depends on corresponding term of $M_0, M_{\tilde{l}}$. Let $M(q)$ denote the q th element of M-sequence. When $M_0(q) = 0, M_{\tilde{l}}(q) = 0$ or $M_0(q) = 1, M_{\tilde{l}}(q) = 1$, $x_q = 1$. When $M_0(q) = 0, M_{\tilde{l}}(q) = 1$, $x_q = e^{j\Delta\phi^M}$. When $M_0(q) = 1, M_{\tilde{l}}(q) = 0$, $x_q = e^{-j\Delta\phi^M}$. Assuming for M_0 and $M_{\tilde{l}}$, the number of four cases above is c_1, c_2, c_3, c_4 , the following 6 equations can be obtained owing to the balance property of M-sequence: $c_1 + c_4 = \frac{L+1}{2}, c_2 + c_3 = \frac{L-1}{2}, c_1 + c_3 = \frac{L+1}{2}, c_2 + c_4 = \frac{L-1}{2}, c_3 + c_4 = \frac{L+1}{2}, c_1 + c_2 = \frac{L-1}{2}$. By solving these equations, we can get $c_1 = \frac{L+1}{4}, c_2 = \frac{L-3}{4}$, and $c_3 = c_4 = \frac{L+1}{4}$. Based on above results, we get

$$\begin{aligned} \frac{1}{L} \sum_{q=0}^{L-1} d_q d_{(q+\tilde{l}) \bmod (L-1)}^* &= \frac{1}{L} \left(\frac{L-1}{2} e^{j0} + \frac{L+1}{4} e^{j\Delta\phi_{\max}^M} + \frac{L+1}{4} e^{j\Delta\phi_{\min}^M} \right) \\ &= \frac{1}{L} \left(\frac{L-1}{2} + \frac{L+1}{2} \cos \Delta\phi^M \right). \end{aligned} \quad (11)$$

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