

Performance Analysis and Waveform Optimization of Integrated FD-MIMO Radar-Communication Systems

Xufeng Zhou, Lan Tang[✉], *Member, IEEE*, Yechao Bai[✉], and Ying-Chang Liang[✉], *Fellow, IEEE*

Abstract—Multiple-input multiple-output (MIMO) has been used in wireless communications to increase data rates via multiplexing or improve performance via diversity. Additionally, in radar systems, MIMO promises improved parameter identifiability and target resolution. Frequency diverse (FD)-MIMO radar can effectively distinguish targets that are closely spaced in the same angle cell by exploiting its range-angle-dependent transmit beamform. Therefore, in this paper, we first propose an integrated waveform design by embedding weighted, phase-modulated communication signals in the FD-MIMO radar waveform. Then, we derive the Cramer-Rao lower bound (CRLB) of the location estimation and analyze the communication performance achieved when the communication receiver extracts information from the integrated signals. Next, transmit beamforming is optimized to achieve the best tradeoff between the radar and communication performances of the system. Due to the nonconvexity of the optimization problem, we apply sequential parametric convex approximation (SPCA) and semidefinite relaxation (SDR) methods to solve the problem. The simulation results reveal the effects of some parameters, such as the frequency interval, symbol period, and communication symbol sequence, on radar and communication performance and demonstrate the tradeoff between radar and communication obtained by optimizing the transmit beamforming vector.

Index Terms—Integrated waveform design, FD-MIMO, Cramér-Rao lower bound, optimization of transmit beamforming.

I. INTRODUCTION

DUE to the radio frequency and space resource shortages on small mobile platforms, the concept of an integrated radar and communication system, which provides dual functions on a single hardware platform with a single waveform, has attracted increasing attention [1]–[3]. Similarities between

radar and communication in signal processing, hardware platforms and system frameworks make it possible to realize the integration of radar and communication.

Multiple-input multiple-output (MIMO) technology has been successfully applied to both wireless communications and radar systems. In communication systems, MIMO is used to improve spectral efficiency through multiplexing or diversity. Furthermore, compared with phased-array radar, an MIMO radar system can transmit multiple probing signals, which enables flexible beam pattern design as well as improved target detection and higher resolution. Waveform optimization has been shown to be a unique advantage of MIMO radar. The probing signal vector transmitted from each antenna can be designed to approximate a desired transmit beam pattern. When the target locations are unknown, orthogonal waveforms are used for MIMO probing [4]. Otherwise, a covariance matrix is designed by making the beam pattern match the desired transmit beam pattern over the area of interest [5].

Therefore, it is significant to investigate an integrated radar-communication system based on MIMO technology. The main MIMO waveform design methods for integrated radar-communication functions can be classified into two categories. The first class involves embedding information symbols in radar waveforms. In this case, a communication function is implemented on the premise of ensuring radar performance. Both [6] and [7] investigated how to achieve the tradeoff between the primary radar and the secondary communication operations. [6] proposed phase modulation-based and sidelobe control-based information embedding in dual-function radar communications. In [7], two novel signaling strategies were proposed to embed information bits in each radar pulse by exploiting waveform diversity in tandem with sidelobe control. When MIMO radar employed orthogonal waveforms, the communication function can be realized by embedding one phase-shift keying (PSK) communication symbol in each orthogonal radar waveform [8]. The analysis in [9] showed that embedding PSK symbols in orthogonal frequency-hopping waveforms yields a reduction in the sidelobe peaks of the ambiguity function. [10] proposed an information embedding scheme based on shuffling orthogonal MIMO radar waveforms. Another class employs communication signals as the probing radar waveforms. In [11], the communication beamformer was optimized so that the beam pattern satisfied the performance requirements of both radar and communication.

Manuscript received February 12, 2021; revised April 22, 2021; accepted May 26, 2021. Date of publication June 9, 2021; date of current version November 11, 2021. This work was supported in part by the National Key Research and Development Program of China under Grant 2019YFE0113200 and in part by the National Natural Science Foundation of China under Grant 62072229, Grant 62071220, and Grant U1936201. The associate editor coordinating the review of this article and approving it for publication was X. Tao. (*Corresponding author: Lan Tang.*)

Xufeng Zhou, Lan Tang, and Yechao Bai are with the School of Electronic Science and Engineering, Nanjing University, Nanjing 210093, China (e-mail: tanglan@nju.edu.cn).

Ying-Chang Liang is with the Center for Intelligent Networking and Communication (CINC), University of Electronic Science and Technology of China, Chengdu 610054, China.

Color versions of one or more figures in this article are available at <https://doi.org/10.1109/TWC.2021.3085250>.

Digital Object Identifier 10.1109/TWC.2021.3085250

[12] optimized the transmit signal matrix by minimizing the multiuser interference under MIMO radar-specific constraints. The most popular communication signal for automotive joint radar-communication (JRC) is orthogonal frequency division multiplexing (OFDM). In [13], the multiplexing strategies, statistical bounds and low-complexity receiver processing for both phase-modulated continuous wave and OFDM waveforms were investigated in the bi-static JRC system. In both [14] and [15], adaptive preamble design schemes were proposed to achieve a trade-off between radar parameters' estimation accuracy and communication rate.

Although MIMO radar possesses many advantages, it cannot distinguish between potential targets and interference when the target and the interference have the same angle but different ranges because conventional MIMO radar has angle-dependent beam patterns [16]. Frequency diverse (FD) techniques have been proposed to overcome the problem mentioned above. Different from conventional phased-array and MIMO radar, FD radar is a new concept that was proposed in [17], [18]. The frequency of the waveform radiated from each element is incremented by a small amount (Δf) from element to element. The frequency incrementation is small compared to the carrier frequency so that we can obtain a range-angle-dependent beampattern, which is favorable for target detection in the presence of interference. The CRLB, mean square error (MSE) and resolution performance of the range-angle estimation for FD-MIMO radar were discussed in [19], which verified that FD-MIMO radar outperforms conventional MIMO radar in terms of both range-angle estimation and resolution performance.

In [20], the authors considered a dual-function FD-MIMO radar-communication system employing Costas signal waveforms that embedded phase-modulated communication information into the Costas coding waveforms. [21] explored time-modulated frequency diverse MIMO (TMFD-MIMO) for integrated radar and communications, where the communication information bits were embedded via the spread sequence technique during each radar pulse. Both [20] and [21] did not consider how to optimize transmit beamforming to obtain the best tradeoff between the radar and communication performances of the system.

In this paper, we consider an integrated FD-MIMO radar-communication system for a short-distance probing scene. We first design an integrated waveform based on FD-MIMO by embedding weighted phase-modulated signals in orthogonal FD waveforms or Costas frequency waveforms. Then, the CRLB of multitarget location estimation is derived by considering different transmission delays. Furthermore, for the communication receiver, the capacity and achievable transmission rate are analyzed. From the analytical results of the CRLB and BER, we can see that the transmission weight vector affects both the radar and communication performances of the system. Therefore, the weight vector of the integrated signal is optimized by maximizing the capacity of the communication receiver under the constraint of the CRLB. Because the optimization problem is nonconvex, we adopt the sequential parametric convex approximation (SPCA) method and semidefinite relaxation (SDR) to solve the problem.

Compared with those of existing works, the main contributions of this paper are as follows:

- An integrated waveform design method is proposed by embedding weighted, phase-modulated signals in an FD-MIMO radar waveform. Both the CRLB of location estimation and communication performance metrics, such as the capacity and achievable transmission rate, are analyzed.
- To achieve the best tradeoff between location estimation and communication performance, we optimize the transmission weight vector by maximizing the capacity of the communication receiver under the constraint of estimation performance. The SPCA and SDR are applied to solve the nonconvex optimization problem.

The rest of the paper is organized as follows: Section II introduces the system model and waveform design of the integrated FD-MIMO system. In Section III, we analyze the CRLB of location estimation and the communication performance of the system. In Section IV, the transmit beamforming vector is optimized to achieve the best tradeoff between radar and communication performances of the system. Section V presents the simulation results, and finally, conclusions are drawn in Section VI.

II. SYSTEM MODEL AND WAVEFORM DESIGN

A. System Model

We consider an integrated FD-MIMO radar-communication system, which consists of one dual-function transmitter, one radar receiver array and multiple targets equipped with communication receivers. We assume that the transmitter and radar receiver have M and N antennas, respectively, and each communication receiver has a single antenna. Furthermore, we suppose that the dual-function transmitter and radar receiver are co-located so that a target in the far field is at the same spatial angle with respect to both the transmitter and radar receiver.

The transmitter periodically transmits pulses carrying information to perform the data transmission and radar sensing. We assume that in each pulse period, the transmitter only conveys information to one target. Therefore, by transmitting dual-function signals, information can be sent to the specific target, and the positions of all targets are estimated by echo signals at the radar receiver. Furthermore, we assume that the communication between the transmitter and the communication target is done through line of sight transmission and that the communication channel can be perfectly estimated.

B. Waveform Design for Integrated FD-MIMO Signals

To provide the range-angle-dependent beampattern for distinguishing between potential targets that have the same angle but different ranges, we adopt the multi-antenna FD transmitter. To embed communication information into FD-MIMO radar waveforms, we introduce M_p -ary PSK in each transmit waveform. Supposing that L symbols are transmitted during each pulse period T , the integrated signals emitted from the m th antenna are expressed as

$$x_m(t) = \text{Re}[w_m s(t) \exp(j2\pi f_m t)], \quad m = 1, 2, \dots, M, \quad (1)$$

where $s(t) = \sum_{l=0}^{L-1} \exp(j\varphi_l)g(t - lT_p)$, $\varphi_l \in [0, \frac{2\pi}{M_p}, \dots, \frac{2\pi(M_p-1)}{M_p}]$ is the phase of the l th M_p -PSK signal, $g(t)$ is a rectangular pulse with amplitude A and duration T_p , and $\{w_m\}_{m=1}^M$ are transmit beamforming weights with $\sum_{m=1}^M |w_m|^2 = 1$. In (1), different array elements transmit distinct frequencies that are equally spaced by Δf . That is, $f_m = f_0 + c(m)\Delta f$, where f_0 denotes the carrier frequency. In the basic FD scheme, each array element radiates an incremental carrier frequency so that $c(m) = m$. In the FD scheme with Costas frequency coding [20], $\{c(m)\}_{m=1}^M$ are Costas codes. For example, for $M = 10$, $\{c(m)\}_{m=1}^M = \{1, 2, 4, 8, 5, 10, 9, 7, 3, 6\}$. The transmission frequencies in different antennas satisfy

$$\int_0^T \exp\{j2\pi f_m t\} \cdot \exp\{-j2\pi f_n t\} dt = 0. \quad (2)$$

The minimum and maximum frequency intervals satisfying (2) are $\Delta f = \frac{1}{T}$ and $\Delta f = \frac{1}{T_p}$, respectively. When $\Delta f = \frac{1}{T_p}$, M frequencies are also orthogonal in each T_p . For a given bandwidth W and pulse period T , when $\Delta f = \frac{1}{T}$, $\frac{M-1}{T} + \frac{1}{T_p} = W$ and $L = WT - M + 1$, and when $\Delta f = \frac{1}{T_p}$, $\frac{M}{T_p} = W$ and $L = \frac{WT}{M}$. Therefore, when $\Delta f = \frac{1}{T}$, additional communication signals can be embedded into the transmission waveform.

We consider a short-distance probing scenario in which the transmission delay is smaller than T so that the radar receiver must continually receive echo signals, even when the transmitter is transmitting information. Although, partial transmission signals in this mode will leak to the radar receiver, the transmission leakage can be restrained by transmission leakage cancellation techniques, such as antenna isolation and adaptive interference cancellation. Therefore, in this paper, the transmission leakage is ignored. Suppose that the pulse repetition period is equal to $2T$ and that there are K targets located in the probing area. The baseband signals received by the n th radar receiver antenna are

$$y_n(t) = \sum_{k=1}^K \sum_{m=1}^M \varepsilon_{mnk} x_m(t - \tau_{mnk}) + n(t) \quad (3)$$

where $\tau_{mnk} = \frac{2r_k - (m+n)d \sin \theta_k}{c}$, r_k and θ_k are the distance and angle, respectively, between the transmitter and the k th target, d is the antenna spacing, $\varepsilon_{mnk} \propto \frac{1}{r_k^2}$ denotes the propagation attenuations related to the k th target, and $n(t)$ is additive white Gaussian noise (AWGN) with zero mean and variance σ^2 .

Each $y_n(t)$ is passed through M correlators, and the output of the $\tilde{m}$ th correlator is

$$\begin{aligned} y_{\tilde{m}n} &= \int_0^T y_n(t) s^*(t) \exp\{-j2\pi c(\tilde{m})\Delta f t\} dt \\ &= \sum_{k=1}^K x_{\tilde{m}nk} + n_{\tilde{m}n} \end{aligned} \quad (4)$$

where

$$\begin{aligned} x_{\tilde{m}nk} &= \sum_{m=1}^M w_m \varepsilon_{mnk} \exp\{-j2\pi c(m)\Delta f \tau_{mnk}\} \lambda(\tau_{mnk}, m, \tilde{m}), \\ \lambda(\tau_{mnk}, m, \tilde{m}) &= \sum_{l=0}^{L-1} \sum_{\tilde{l}=0}^{L-1} \exp(j(\varphi_l - \varphi_{\tilde{l}})) \int_0^T g(t - lT_p - \tau_{mnk}) \\ &\quad g(t - \tilde{l}T_p) \exp(j2\pi(c(m) - c(\tilde{m}))\Delta f t) dt \end{aligned} \quad (5)$$

and $n_{\tilde{m}n}$ is AWGN with zero mean and variance $\sigma_w^2 = \sigma^2 A^2 T$. Through calculations, (5) can be represented as

$$\lambda(\tau_{mnk}, m, \tilde{m}) = \begin{cases} \sum_{\tilde{l}=l}^{L-1} \lambda_{\tilde{l}-l} \exp(j(\varphi_{\tilde{l}-l} - \varphi_{\tilde{l}})) \int_{\tau_{mnk}}^{(l+1)T_p} f(t) dt \\ + \sum_{\tilde{l}=l+1}^{L-1} \lambda_{\tilde{l}-l} \exp(j(\varphi_{\tilde{l}-l-1} - \varphi_{\tilde{l}})) \int_{lT_p}^{\tau_{mnk}} f(t) dt, & T_p < \tau_{mnk} \leq (l+1)T_p, 0 \leq l \leq L-2 \\ A^2 \exp(j(\varphi_0 - \varphi_{L-1})) \int_{\tau_{mnk}}^{LT_p} f(t) dt, & (L-1)T_p < \tau_{mnk} \leq LT_p \\ 0, & \text{others} \end{cases} \quad (6)$$

where $\lambda_{\tilde{l}-l} = A^2 \exp(j2\pi(c(m) - c(\tilde{m}))\Delta f(\tilde{l}-l)T_p)$, $f(t) = \exp(j2\pi(c(m) - c(\tilde{m}))\Delta f t)$.

When $\tau_{mnk} \ll T$, $s(t) \approx s(t - \tau_{mnk})$ such that $x_{\tilde{m}nk}$ in (4) can be simplified as $x_{\tilde{m}nk} = w_{\tilde{m}} \varepsilon_{\tilde{m}nk} A^2 T \exp\{-j2\pi c(\tilde{m})\Delta f \tau_{mnk}\}$. When T is small and the targets are far from the transmitter, the condition $\tau_{mnk} \ll T$ may be violated. Therefore, in (4), we consider the more general case with $0 < \tau_{mnk} \leq T$ and regard $\tau_{mnk} \ll T$ as a special case. Additionally, it is observed from (5) that when $\tau_{mnk} \ll T$, the communication symbol sequence $[\exp(j\varphi_0), \dots, \exp(j\varphi_{L-1})]$ has no effect on $y_n(t)$. Otherwise, the communication symbol sequence affects $y_n(t)$ and the radar performance.

For the communication receiver located at (r, θ) , the received equivalent baseband signal in the l th symbol period is

$$\begin{aligned} r(t) &= \sum_{m=1}^M A h_m w_m \exp(j\varphi_l) \exp(j2\pi c(m)\Delta f t) + z(t), \\ lT_p &\leq t \leq (l+1)T_p, \end{aligned} \quad (7)$$

where $h_m = \varepsilon \exp\{j2\pi f_m \frac{r - md \sin \theta}{c}\}$ ($\varepsilon \propto \frac{1}{r}$) is the channel coefficient between the m th transmission antenna and the communication receiver and $z(t)$ is AWGN with mean zero and variance σ^2 .

III. PERFORMANCE ANALYSIS OF INTEGRATED FD-MIMO SIGNALS

A. CRLB of Integrated FD-MIMO Signals

In this subsection, we analyze the CRLB of location estimation based on (4). First, the unknown parameters can be grouped as $\gamma = [\tau_1, \dots, \tau_K, \varepsilon]$

and $\mathbf{u} = [(\theta_1, r_1), \dots, (\theta_K, r_K), \varepsilon]^T$, where $\varepsilon = [\varepsilon_{111}, \varepsilon_{211}, \dots, \varepsilon_{MKN}]$ and $\tau_k = [\tau_{11k}, \tau_{21k}, \dots, \tau_{MKNk}]$. As the signal in (4) is a function of $\{\tau_k\}_{k=1}^K$, we first compute the Fisher information matrix (FIM) $\mathbf{J}(\gamma)$ with respect to γ and then use the chain rule to evaluate

$$\mathbf{J}(\mathbf{u}) = \mathbf{Q} \mathbf{J}(\gamma) \mathbf{Q}^T \quad (8)$$

where $\mathbf{Q}$ is the Jacobian matrix. $\mathbf{Q}$ can be calculated as

$$\mathbf{Q} = \frac{\partial \gamma}{\partial \mathbf{u}} = \begin{bmatrix} \mathbf{x}_1 & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{z}_1 & \dots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \dots & \mathbf{x}_K & \mathbf{0} \\ \mathbf{0} & \dots & \mathbf{z}_K & \mathbf{0} \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{I} \end{bmatrix} \quad (9)$$

where $[\mathbf{x}_k]_{(n-1)M+m} = \frac{-(m+n)d \cos \theta_k}{c}$, $[\mathbf{z}_k]_{(n-1)M+m} = \frac{2}{c}$, $m = 1, \dots, M$, $n = 1, \dots, N$, and $\mathbf{I}$ is a $MNK \times MNK$ unit matrix. We arrange all outputs of the MN correlators into a vector $\mathbf{y} = [y_{11}, y_{21}, \dots, y_{MKN}]^T$. Because $\mathbf{y}$ is a Gaussian vector with mean $\boldsymbol{\mu} = [\sum_{k=1}^K x_{11k}, \sum_{k=1}^K x_{21k}, \dots, \sum_{k=1}^K x_{MKNk}]^T$ and a covariance matrix of $\sigma_w^2 \mathbf{I}$, the (i, j) th element in $\mathbf{J}(\gamma)$ can be calculated as

$$[\mathbf{J}(\gamma)]_{ij} = 2\text{Re}[\frac{1}{\sigma_w^2} \frac{\partial \boldsymbol{\mu}}{\partial \gamma_i} \frac{\partial \boldsymbol{\mu}}{\partial \gamma_j}] \quad (10)$$

where γ_i is the i th element of γ . Based on (8)-(10), we can obtain the following FIM for vector $\mathbf{u}$:

$$\mathbf{J}(\mathbf{u}) = \begin{bmatrix} \mathbf{J}_1^{(1)} & \mathbf{J}_2^{(1)} \dots & \mathbf{J}_K^{(1)} & \mathbf{0} \\ \mathbf{J}_1^{(2)} & \mathbf{J}_2^{(2)} \dots & \mathbf{J}_K^{(2)} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{J}_1^{(K)} & \mathbf{J}_2^{(K)} \dots & \mathbf{J}_K^{(K)} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \dots & \mathbf{0} & \mathbf{C} \end{bmatrix} \quad (11)$$

where $\mathbf{J}_k^{(k')}$ is a 2×2 matrix with

$$[\mathbf{J}_k^{(k')}]_{1,1} = \frac{1}{\sigma_w^2} \mathbf{w}(\mathbf{G}_a^{kk'} + \{\mathbf{G}_a^{kk'}\}^H) \mathbf{w}^H, \quad (12)$$

$$[\mathbf{J}_k^{(k')}]_{1,2} = \frac{1}{\sigma_w^2} \mathbf{w}(\mathbf{G}_b^{kk'} + \{\mathbf{G}_b^{kk'}\}^H) \mathbf{w}^H, \quad (13)$$

$$[\mathbf{J}_k^{(k')}]_{2,1} = \frac{1}{\sigma_w^2} \mathbf{w}(\mathbf{G}_c^{kk'} + \{\mathbf{G}_c^{kk'}\}^H) \mathbf{w}^H, \text{ and} \quad (14)$$

$$[\mathbf{J}_k^{(k')}]_{2,2} = \frac{1}{\sigma_w^2} \mathbf{w}(\mathbf{G}_d^{kk'} + \{\mathbf{G}_d^{kk'}\}^H) \mathbf{w}^H. \quad (15)$$

The derivation of $\mathbf{J}(\mathbf{u})$ and expressions for $\mathbf{G}_a^{kk'}$, $\mathbf{G}_b^{kk'}$, $\mathbf{G}_c^{kk'}$, and $\mathbf{G}_d^{kk'}$ are given in the appendix.

Let $\hat{\mathbf{u}} = [(\theta_1, r_1), \dots, (\theta_K, r_K)]$. The CRLB matrix of $\hat{\mathbf{u}}$ is

$$\mathbf{C}_{\hat{\mathbf{u}}}(\mathbf{w}) = \begin{bmatrix} \mathbf{J}_1^{(1)} & \mathbf{J}_2^{(1)} \dots & \mathbf{J}_K^{(1)} \\ \mathbf{J}_1^{(2)} & \mathbf{J}_2^{(2)} \dots & \mathbf{J}_K^{(2)} \\ \vdots & \vdots & \vdots \\ \mathbf{J}_1^{(K)} & \mathbf{J}_2^{(K)} \dots & \mathbf{J}_K^{(K)} \end{bmatrix}^{-1}. \quad (16)$$

From (16), we can see that due to the nonprincipal diagonal matrices representing the couplings among targets, it is difficult to obtain the closed-form expression for each diagonal

element of $\mathbf{C}_{\hat{\mathbf{u}}}(\mathbf{w})$. However, we can obtain the lower bound of each diagonal element of $\mathbf{C}_{\hat{\mathbf{u}}}(\mathbf{w})$ by utilizing the following property [22]:

$$[\mathbf{J}_k^{(k)}]_{ii}^{-1} \leq [\mathbf{C}_{\hat{\mathbf{u}}}(\mathbf{w})]_{2(k-1)+i, 2(k-1)+i}, \quad i = 1, 2. \quad (17)$$

When targets are far apart and the aperture of the receiver antenna is large, elements in the nonprincipal diagonal matrix have slight effects on the diagonal elements of $\mathbf{C}_{\hat{\mathbf{u}}}(\mathbf{w})$. In this case, the lower bound in (17) is tight. Based on (17), we can obtain the CRLB matrix of the k th target location:

$$\begin{aligned} \mathbf{C}_k(\mathbf{w}) &= \sigma_w^2 \left[\mathbf{w}(\mathbf{G}_a^{kk} + \{\mathbf{G}_a^{kk}\}^H) \mathbf{w}^H \quad \mathbf{w}(\mathbf{G}_b^{kk} + \{\mathbf{G}_b^{kk}\}^H) \mathbf{w}^H \right. \\ &\quad \left. \mathbf{w}(\mathbf{G}_c^{kk} + \{\mathbf{G}_c^{kk}\}^H) \mathbf{w}^H \quad \mathbf{w}(\mathbf{G}_d^{kk} + \{\mathbf{G}_d^{kk}\}^H) \mathbf{w}^H \right]^{-1} \end{aligned} \quad (18)$$

Finally, we consider the simplified case with $\tau_{mnk} \ll T$. In this case, $\mathbf{J}(\gamma)$ in (43) becomes a diagonal matrix. That is, in (44), the $\mathbf{A}_k^{(k)}$ s are diagonal matrices and $\mathbf{A}_k^{(k')} = \mathbf{0}, \forall k \neq k'$. Hence, $\mathbf{A}_k^{nk}$ in (45) is also a diagonal matrix with $a_{kmm}^{nk} = 4|\varepsilon_{mnk}|^2 A^4 T^2 \pi^2 c^2(m) \Delta f^2$. Then, $\mathbf{J}(\mathbf{u})$ can be calculated as

$$\mathbf{J}(\mathbf{u}) = \begin{bmatrix} \mathbf{J}_1^{(1)} & \mathbf{0} \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_2^{(2)} \dots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} \dots & \mathbf{J}_K^{(K)} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \dots & \mathbf{0} & \mathbf{C} \end{bmatrix} \quad (19)$$

where $\mathbf{J}_k^{(k)}$ is a 2×2 matrix with

$$[\mathbf{J}_k^{(k)}]_{1,1} = \frac{2}{\sigma_w^2} \mathbf{w} \mathbf{G}_a^{kk} \mathbf{w}^H \quad (20)$$

$$[\mathbf{J}_k^{(k)}]_{1,2} = [\mathbf{J}_k^{(k)}]_{2,1} = \frac{2}{\sigma_w^2} \mathbf{w} \mathbf{G}_b^{kk} \mathbf{w}^H \quad (21)$$

$$[\mathbf{J}_k^{(k)}]_{2,2} = \frac{2}{\sigma_w^2} \mathbf{w} \mathbf{G}_d^{kk} \mathbf{w}^H \quad (22)$$

where $\mathbf{G}_a^{kk}$, $\mathbf{G}_b^{kk}$, and $\mathbf{G}_d^{kk}$ are diagonal matrices with diagonal elements $[\mathbf{G}_a^{kk}]_{m,m} = \sum_{n=1}^N \{a_{kmm}^{nk} [\mathbf{x}_k]_{(n-1)M+m}^2\}$, $[\mathbf{G}_b^{kk}]_{m,m} = \sum_{n=1}^N \frac{2}{c} \{a_{kmm}^{nk} [\mathbf{x}_k]_{(n-1)M+m}\}$ and $[\mathbf{G}_d^{kk}]_{m,m} = \sum_{n=1}^N \frac{4}{c^2} \{a_{kmm}^{nk}\}$, respectively. The CRLB matrix of the k th target $\mathbf{C}_k(\mathbf{w}) = [\mathbf{J}_k^{(k)}]^{-1}$ has a similar form as that of (18). The difference is that in this simplified case, the CRLB is only related to the amplitudes of $\{w_m\}_{m=1}^M$ and is independent of their phases.

B. Communication Performance of the Integrated FD-MIMO System

In this subsection, we analyze the communication rate and BER performance of the integrated system based on (7). During the l th symbol period, by correlating the received

signals $r(t)$ in (7) with M different correlators, we can obtain

$$\begin{aligned} r_{m'}^l &= \int_{lT_p}^{(l+1)T_p} r(t) \exp(-j2\pi c(m')\Delta f t) dt \\ &= A \exp(j\varphi_l) \sum_{m=1}^M h_m w_m \rho_{mm'}^l + z_{m'}^l, 1 \leq m' \leq M \end{aligned} \quad (23)$$

where $\rho_{mm'}^l = \int_{lT_p}^{(l+1)T_p} \exp(j2\pi(c(m) - c(m'))\Delta f t) dt$ and $z_{m'}^l = \int_{lT_p}^{(l+1)T_p} z(t) \exp\{-j2\pi c(m')\Delta f t\} dt$ is AWGN with variance $\sigma^2 T_p$. Combining all $r_{m'}^l$ s together, we obtain

$$r^l = \sum_{m'=1}^M r_{m'}^l = A \exp(j\varphi_l) \mathbf{w} \tilde{\mathbf{h}}^l + \sum_{m'=1}^M z_{m'}^l \quad (24)$$

where $[\tilde{\mathbf{h}}^l]_m = h_m \sum_{m'=1}^M \rho_{mm'}^l$. Note that the $\{z_{m'}^l\}_{m'=1}^M$ s are correlated Gaussian variables. Therefore, $E[\sum_{m=1}^M \sum_{m'=1}^M z_{m'}^l z_{m'}^{l*}] = \sigma^2 \sum_{m=1}^M \sum_{m'=1}^M \rho_{mm'}^l$. From (24), we can calculate that the SNR of the l th symbol is

$$SNR_l = \frac{A^2 |\mathbf{w} \tilde{\mathbf{h}}^l|^2}{\sigma^2 \sum_{m=1}^M \sum_{m'=1}^M \rho_{mm'}^l}, \quad 1 \leq l \leq L. \quad (25)$$

Based on (25), during a pulse period, the channel capacity and the information bits successfully received by the communication receiver are given by

$$C = \sum_{l=1}^L \log(1 + SNR_l), \quad (26)$$

and

$$R = \sum_{l=1}^L \log(M_p)(1 - BER_l) \quad (27)$$

respectively, where $BER_l \approx 2Q(\sqrt{2SNR_l} \sin(\frac{\pi}{M_p}))$ is the BER of the l th symbol and $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-y^2/2} dy$ is the Gaussian Q-function.

When $\Delta f = \frac{1}{T_p}$, $\rho_{mm'}^l = T_p \delta(m - m')$ and the received SNR is

$$SNR_l = \frac{A^2 T_p |\mathbf{w} \mathbf{h}|^2}{M \sigma^2}, \quad \forall l, \quad (28)$$

where $\mathbf{h} = [h_1, \dots, h_M]^T$. Notably, in this case, the maximum SNR can be achieved via maximum ratio transmission. That is, $\mathbf{w}^* = \frac{\mathbf{h}^H}{\|\mathbf{h}\|}$. By substituting $SNR_l = \frac{A^2 T_p \|\mathbf{h}\|^2}{M \sigma^2}$ into (26) and (27), the channel capacity and achievable transmission rate are derived.

IV. OPTIMIZATION OF THE TRANSMIT BEAMFORMING VECTOR FOR THE INTEGRATED FD-MIMO SYSTEM

From the expressions for the CRLB and transmission rate, we can see that the beamforming vector $\mathbf{w}$ affects both the radar and communication performances of the integrated system. Therefore, we optimize $\mathbf{w}$ by considering both the estimation and communication performances. We re-express (18) as

$$\mathbf{C}_k(\mathbf{w}) = \sigma_w^2 \begin{bmatrix} \mathbf{W}(\cdot) \mathbf{g}_a^{kk} & \mathbf{W}(\cdot) \mathbf{g}_b^{kk} \\ \mathbf{W}(\cdot) \mathbf{g}_c^{kk} & \mathbf{W}(\cdot) \mathbf{g}_d^{kk} \end{bmatrix}^{-1} \quad (29)$$

where $\mathbf{g}_a^{kk}, \mathbf{g}_b^{kk}, \mathbf{g}_c^{kk}$ and $\mathbf{g}_d^{kk}$ are vectors obtained by vertically stacking the columns of $\mathbf{G}_a^{kk} + \{\mathbf{G}_a^{kk}\}^H, \mathbf{G}_b^{kk} + \{\mathbf{G}_b^{kk}\}^H, \mathbf{G}_c^{kk} + \{\mathbf{G}_c^{kk}\}^H$ and $\mathbf{G}_d^{kk} + \{\mathbf{G}_d^{kk}\}^H$, respectively, and $\mathbf{W} = \mathbf{w}^H \mathbf{w}$ and $\mathbf{W}(\cdot)$ are the vectors obtained by vectorizing $\mathbf{W}$ row-wise.

To unify the dimension of distance error and angle error, the scaled CRLB of the k th target can be calculated as

$$CRLB_{\theta_k, r_k} = \frac{\mathbf{W}(\cdot) \mathbf{b}_k}{\mathbf{W}(\cdot) \mathbf{A}_k \mathbf{W}(\cdot)^H} \quad (30)$$

where $\mathbf{b}_k = \sigma_w^2 (\mathbf{g}_a^{kk} + \frac{r_k}{\theta_k} \mathbf{g}_d^{kk})$ and $\mathbf{A}_k = \mathbf{g}_a^{kk} \{\mathbf{g}_d^{kk}\}^H - \mathbf{g}_b^{kk} \{\mathbf{g}_c^{kk}\}^H$.

A. Minimizing the CRLB by Optimizing $\mathbf{w}$

In this subsection, we investigate how to minimize $\max_k CRLB_{\theta_k, r_k}$ by ignoring the communication performance requirement. Supposing that $\max_k CRLB_{\theta_k, r_k} = \eta$, the CRLB minimization problem can be expressed as the following feasibility problem:

$$\text{find } \mathbf{W} \succeq \mathbf{0} \in \mathbb{H}^{M \times M} \quad \mathbf{W} \quad (31)$$

$$\text{s. t. } \text{tr}(\mathbf{W}) = 1, \quad (31a)$$

$$\mathbf{W}(\cdot) \mathbf{b}_k > 0, \quad k = 1, \dots, K, \quad (31b)$$

$$\mathbf{W}(\cdot) \mathbf{b}_k - \eta \mathbf{W}(\cdot) \mathbf{A}_k \mathbf{W}(\cdot)^H \leq 0, \quad k = 1, \dots, K, \quad (31c)$$

$$\text{Rank}(\mathbf{W}) = 1, \quad (31d)$$

where $\mathbb{H}^{M \times M}$ is a set of $M \times M$ Hermitian matrices.

Let c^* be the minimum value of $\max_k CRLB_{\theta_k, r_k}$. If the feasibility problem in (31) is feasible, we have $c^* < \eta$. Conversely, if the problem is infeasible, we can conclude that $c^* > \eta$. The optimal c^* can be found by updating η with the bisection method [23]. For a given η , the difficult part is that the problem in (31) is nonconvex due to its nonconvex constraints. The constraint in (31c) is nonconvex since $\mathbf{A}_k$ is a nonnegative definite matrix. To obtain an approximate solution, we first remove the rank constraint by semidefinite relaxation (SDR) and then use the sequential parametric convex approximation (SPCA) approach in [24] to transform (31) into a convex problem. The basic idea of SPCA is that during each iteration, the nonconvex constraint is replaced by its convex approximate upper function. Define $f_k(\mathbf{W}(\cdot)) = -\mathbf{W}(\cdot) \mathbf{A}_k \mathbf{W}(\cdot)^H$. To find the convex upper function of $f_k(\mathbf{W}(\cdot))$, we need the following result.

Proposition 1 ([25]): Let $f(x), x \in \mathbb{R}^n$ be a (possibly) nonconvex function (assumed to be continuously differentiable) whose gradient satisfies a Lipschitz condition with a constant L . Then, we have

$$f(x) \leq f(x_0) + \Delta f(x_0)(x - x_0)^T + \frac{L}{2} \|x - x_0\|^2, \quad \forall x_0 \in \mathbb{R}^n. \quad (32)$$

When $\mathbf{W}(\cdot) \in \mathbb{R}^{M^2}$ and $\mathbf{A}_k \in \mathbb{R}^{M \times M}$, we can prove the following lemma.

Lemma 1: $f_k(\mathbf{W}(\cdot))$ is a continuously differentiable function whose gradient satisfies a Lipschitz condition with a constant $L = \sqrt{\lambda_{\max}\{(\mathbf{A}_k + \mathbf{A}_k^T)^2\}}$.

Proof: Based on [25], $f_k(\mathbf{W}(\cdot))$ is said to have an L -Lipschitz continuous gradient if for some $L > 0$,

$$\|\nabla f_k(\mathbf{W}_x(\cdot)) - \nabla f_k(\mathbf{W}_y(\cdot))\| \leq L \|\mathbf{W}_x(\cdot) - \mathbf{W}_y(\cdot)\| \quad \forall \mathbf{W}_x(\cdot), \mathbf{W}_y(\cdot) \quad (33)$$

According to the property of the Rayleigh quotient, we can obtain

$$\begin{aligned} \max \frac{\|\nabla f_k(\mathbf{W}_x(\cdot)) - \nabla f_k(\mathbf{W}_y(\cdot))\|^2}{\|\mathbf{W}_x(\cdot) - \mathbf{W}_y(\cdot)\|^2} \\ = \frac{(\mathbf{W}_x(\cdot) - \mathbf{W}_y(\cdot))(\mathbf{A}_k + \mathbf{A}_k^T)^2(\mathbf{W}_x(\cdot) - \mathbf{W}_y(\cdot))^H}{(\mathbf{W}_x(\cdot) - \mathbf{W}_y(\cdot))(\mathbf{W}_x(\cdot) - \mathbf{W}_y(\cdot))^H} \\ = \lambda_{\max}\{(\mathbf{A}_k + \mathbf{A}_k^T)^2\} \end{aligned} \quad (34)$$

where $\lambda_{\max}\{(\mathbf{A}_k + \mathbf{A}_k^T)^2\}$ denotes the maximum eigenvalue of $(\mathbf{A}_k + \mathbf{A}_k^T)^2$. Therefore, when $L = \sqrt{\lambda_{\max}\{(\mathbf{A}_k + \mathbf{A}_k^T)^2\}}$, (33) holds. $\square$

In the simplified case with $\tau_{mnk} \ll T$, $\mathbf{W}(\cdot) \in \mathbb{R}^{M^2}$. Therefore, for a given $\mathbf{W}_i(\cdot) \in \mathbb{R}^{M^2}$, we can obtain the following upper approximation of $f_k(\mathbf{W}(\cdot))$ based on **Proposition 1** and **Lemma 1**:

$$\begin{aligned} F_k(\mathbf{W}(\cdot), \mathbf{W}_i(\cdot)) &= f_k(\mathbf{W}_i(\cdot)) + \mathbf{W}_i(\cdot)(\mathbf{A}_k + \mathbf{A}_k^T)(\mathbf{W}(\cdot) \\ &\quad - \mathbf{W}_i(\cdot))^T + \frac{L}{2} \|\mathbf{W}(\cdot) - \mathbf{W}_i(\cdot)\|^2. \end{aligned} \quad (35)$$

However, in the general case with $\tau_{mnk} < T$, $\mathbf{W}(\cdot) \in \mathbb{C}^{M^2}$ such that **Proposition 1** and **Lemma 1** cannot be applied to obtain the upper bound function of $f_k(\mathbf{W}(\cdot))$ directly. Let $\mathbf{W}_R(\cdot)$ and $\mathbf{W}_L(\cdot)$ denote the real part and imaginary part of $\mathbf{W}(\cdot)$, and let $\mathbf{A}_k^R(\cdot)$ and $\mathbf{A}_k^L(\cdot)$ denote the real part and imaginary part of $\mathbf{A}_k(\cdot)$, respectively. $f_k(\mathbf{W}(\cdot))$ is re-expressed as

$$\begin{aligned} f_k(\mathbf{W}(\cdot)) &= (\mathbf{W}_R(\cdot) + \mathbf{W}_L(\cdot)j)(\mathbf{A}_k^R + \mathbf{A}_k^L j) \\ &\quad \times (\mathbf{W}_R(\cdot) - \mathbf{W}_L(\cdot)j)^T \\ &= \mathbf{W}_R(\cdot)\mathbf{A}_k^R\mathbf{W}_R(\cdot)^T + \mathbf{W}_R(\cdot)\mathbf{A}_k^L\mathbf{W}_L(\cdot)^T \\ &\quad + \mathbf{W}_L(\cdot)\mathbf{A}_k^R\mathbf{W}_L(\cdot)^T - \mathbf{W}_L(\cdot)\mathbf{A}_k^L\mathbf{W}_R(\cdot)^T \\ &= \tilde{\mathbf{W}}(\cdot)\tilde{\mathbf{A}}_k\tilde{\mathbf{W}}(\cdot)^T \end{aligned} \quad (36)$$

where $\tilde{\mathbf{W}}(\cdot) = [\mathbf{W}_R(\cdot) \ \mathbf{W}_L(\cdot)]$ and $\tilde{\mathbf{A}}_k = \begin{bmatrix} \mathbf{A}_k^R & \mathbf{A}_k^L \\ -\mathbf{A}_k^L & \mathbf{A}_k^R \end{bmatrix}$. Based on (32) and Lemma 1, the convex upper bound function of $f_k(\mathbf{W}(\cdot))$ is given by

$$\begin{aligned} F_k(\mathbf{W}(\cdot), \mathbf{W}_i(\cdot)) &= f_k(\tilde{\mathbf{W}}_i(\cdot)) + \tilde{\mathbf{W}}_i(\cdot)(\tilde{\mathbf{A}}_k + \tilde{\mathbf{A}}_k^T)(\tilde{\mathbf{W}}(\cdot) - \tilde{\mathbf{W}}_i(\cdot))^T \\ &\quad + \frac{\tilde{L}}{2} \|\tilde{\mathbf{W}}(\cdot) - \tilde{\mathbf{W}}_i(\cdot)\|^2 \end{aligned} \quad (37)$$

where $\tilde{\mathbf{W}}_i(\cdot) = [\mathbf{W}_{iR}(\cdot) \ \mathbf{W}_{iL}(\cdot)]$ and $\tilde{L} = \sqrt{\lambda_{\max}\{(\mathbf{A}_k^R + \mathbf{A}_k^{RT})^2\}}$.

Then, the approximate solution to the original problem in (31) can be obtained by iteratively solving the following convex feasibility problem:

$$\text{find } \mathbf{W}_{\succeq 0 \in \mathbb{H}^{M \times M}} \quad \mathbf{W}$$

$$\text{s. t. } \mathbf{W}(\cdot)\mathbf{b}_k + \eta F_k(\mathbf{W}(\cdot), \mathbf{W}_0(\cdot)) \leq 0, \quad k=1, \dots, K, \quad (31a), (31b) \quad (38)$$

where $\mathbf{W}_0(\cdot)$ is a given initial matrix. The iterative algorithm stops if we cannot find a smaller η to make (38) feasible. If the rank of the optimal SDR solution $\mathbf{W}_{SDR}^*$ is larger than 1, we adopt the randomization approach from [26] to extract an approximate solution from $\mathbf{W}_{SDR}^*$. In each random sampling, we generate a random vector $\mathbf{w} \sim \mathcal{CN}(0, \mathbf{W}_{SDR}^*)$ and use it to construct an approximate rank-1 solution. The detailed algorithm is summarized as Algorithm 1.

Algorithm 1 Solution to (31) Based on SDR and SPCA

Initialize $\eta_l, \eta_h, \varepsilon, D$; Ensure that (38) is feasible for η_l and infeasible for η_h .

Repeat

$$\eta = \frac{\eta_l + \eta_h}{2};$$

Choose a starting point $\mathbf{W}_0(\cdot)$ and solve (38) to obtain

$\mathbf{W}_{SDR}^*$;

If {‘solved’}

$$\eta_h = \eta$$

Else

$$\eta_l = \eta$$

End

Until $\{\eta_h - \eta_l \leq \varepsilon\}$

For $d = 1, \dots, D$

Generate $\tilde{\mathbf{w}}_d \sim \mathcal{CN}(0, \mathbf{W}_{SDR}^*)$;

Calculate $\mathbf{w}_d = \frac{\tilde{\mathbf{w}}_d}{\|\mathbf{w}_d\|}$, $\mathbf{W}_d = \mathbf{w}_d^H \mathbf{w}_d$ and

$\max_k \text{CRLB}_{\theta_k, r_k}^d$.

End

$d^* = \arg \min_d \max_k \text{CRLB}_{\theta_k, r_k}^d$.

Output $\mathbf{w} = \mathbf{w}_{d^*}$ and $\min_{d^*} \max_k \text{CRLB}_{\theta_k, r_k}^{d^*}$.

B. Maximize the Channel Capacity With the CRLB Constraint

In this subsection, we investigate how to achieve a tradeoff between the radar and communication performances of the system by optimizing $\mathbf{w}$. We regard the channel capacity C in (26) as the metric for communication performance. Since C is not a concave function of $\mathbf{w}$, $|\mathbf{w}\tilde{\mathbf{h}}^l|^2$ is written as $\mathbf{W}(\cdot)\tilde{\mathbf{h}}^l$, where $\tilde{\mathbf{h}}^l$ is obtained by vertically stacking the columns of $\tilde{\mathbf{h}}^l(\tilde{\mathbf{h}}^l)^H$. According to the composition rule, which can guarantee concavity, C is a concave function of $\mathbf{W}$.

Our goal is to maximize C under the constraint of the CRLB. The optimization problem is formulated as

$$\begin{aligned} \max_{\mathbf{W}_{\succeq 0 \in \mathbb{H}^{M \times M}}} \quad & C \\ \text{s.t.} \quad & (31a) - (31d) \end{aligned} \quad (39)$$

To guarantee that the problem in (39) is feasible, η in (31c) should not be smaller than the optimal CRLB obtained by Algorithm 1. Similar to the problem in (31), (39) is nonconvex due to the constraints in (31c) and (31d). Therefore, we also adopt SDR and SPCA to obtain the approximate solution to (39). Based on SPCA, the approximate solution to (39)

can be obtained by iteratively solving the following convex problem:

$$\begin{aligned} & \max_{\mathbf{W} \succeq 0 \in \mathbb{H}^{M \times M}} C \\ & \text{s.t. } \mathbf{W}(:, \mathbf{b}_k + \eta F_k(\mathbf{W}(:, \mathbf{W}_0(:))), \leq 0, \quad k = 1, \dots, K, \end{aligned} \quad (31a), (31b) \quad (40)$$

where $\mathbf{W}_i(:)$ is a slack variable updated by the optimal solution to (40) in each iteration. The iterative algorithm stops if $|C_i - C_{i-1}| \leq \varepsilon$, where C_i is the objective value in the i th iteration. Finally, the rank-1 approximation of $\mathbf{W}_{SDR}^*$ is obtained by randomization, as shown in Algorithm 1. The overall algorithm is given in Algorithm 2.

Algorithm 2 Solution to (39) Based on SDR and SPCA

Initialize $\eta, \varepsilon, D, \mathbf{W}_0, i = 0, C_0 = 0$;

Repeat

Compute a solution $\mathbf{W}(:)$ for (40) and C_i for a given $\mathbf{W}_i(:)$;

Set $\mathbf{W}_{i+1}(:) = \mathbf{W}(:)$; $i \leftarrow i + 1$.

Until $|C_{i+1} - C_i| < \varepsilon$

$\mathbf{W}_{SDR}^* \leftarrow \mathbf{W}_i$.

For $d = 1, \dots, D$

Generate $\tilde{\mathbf{w}}_d \sim \mathcal{CN}(0, \mathbf{W}_{SDR}^*)$;

Calculate $\mathbf{w}_d = \frac{\tilde{\mathbf{w}}_d}{\|\tilde{\mathbf{w}}_d\|}$ and $\mathbf{W}_d = \mathbf{w}_d \mathbf{w}_d^H$.

If $\max_k \{CRLB_{\theta_k, r_k}(\mathbf{w}_d)\} \leq \eta$

Calculate the objective value C_d .

If $C_d > C_0$

$C_0 \leftarrow C_d$.

$\mathbf{W}_r^* \leftarrow \mathbf{W}_d$.

End

End

End

Output $\mathbf{W}_r^*$ and C_0 .

C. Optimization in the Simplified Case With $\tau_{mnk} \ll T$

Finally, we investigate the optimization of $\mathbf{w}$ in the simplified case with $\tau_{mnk} \ll T$. In this case, the CRLB matrix of the k th target is calculated according to (19)-(22). We can see that the CRLB matrix is only related to $\{|w_m|\}_{m=1}^M$. Since the phases of $\{|w_m|\}_{m=1}^M$ have no effect on the CRLB, we can determine the phases of $\{|w_m|\}_{m=1}^M$ by optimizing the communication performance via the method in Sec. III-B. Specifically, we first solve the following problem by SDR:

$$\begin{aligned} & \max_{\mathbf{W} \succeq 0 \in \mathbb{H}^{M \times M}} C \\ & \text{s.t. } \text{tr}(\mathbf{W}) = 1 \\ & \quad \text{rank}(\mathbf{W}) = 1. \end{aligned} \quad (41)$$

Let $\tilde{\mathbf{w}}$ be the rank-1 approximation of the SDR solution to (41). We denote $\mathbf{w} = [|w_1|, |w_2|, \dots, |w_M|] \mathbf{\Lambda}$, where $\mathbf{\Lambda}$ is a diagonal matrix with $[\mathbf{\Lambda}]_{m,m} = \exp(j \arg([\tilde{\mathbf{w}}]_m))$. Letting $|\mathbf{w}| = [|w_1|, |w_2|, \dots, |w_M|]$, we have $|\mathbf{w} \tilde{\mathbf{h}}|^2 = \bar{\mathbf{W}}(:) \hat{\mathbf{h}}$, where $\bar{\mathbf{W}} = |\mathbf{w}|^H |\mathbf{w}|$ and $\hat{\mathbf{h}}$ is obtained by vertically stacking the columns of $\mathbf{\Lambda} \tilde{\mathbf{h}}^l (\tilde{\mathbf{h}}^l)^H \mathbf{\Lambda}^H$. Based on the above results, (41)

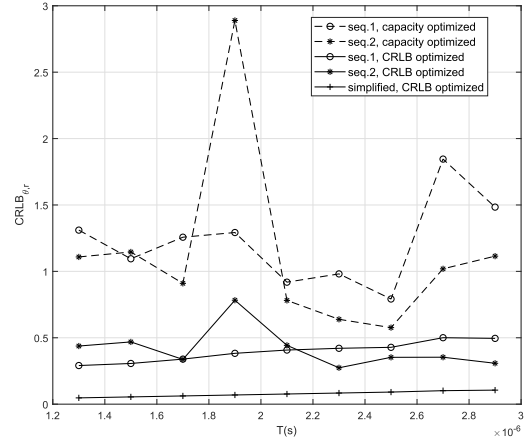


Fig. 1. CRLB vs. T with different communication sequences ($M = N = 10, \Delta f = \frac{1}{T}$).

can be simplified as

$$\begin{aligned} & \max_{\bar{\mathbf{W}} \succeq 0 \in \mathbb{S}^{M \times M}} C \\ & \text{s.t. } \bar{\mathbf{W}}(:, \mathbf{b}_k - \bar{\mathbf{W}}(:, \mathbf{A}_k \bar{\mathbf{W}}(:)^H \eta) \leq 0, \quad k = 1, 2, \dots, K \\ & \quad \bar{\mathbf{W}}(:, \mathbf{b}_k) > 0, \quad k = 1, \dots, K, \\ & \quad \text{tr}(\bar{\mathbf{W}}) = 1 \\ & \quad \text{Rank}(\bar{\mathbf{W}}) = 1 \end{aligned} \quad (42)$$

where $\mathbb{S}^{M \times M}$ is a set of $M \times M$ real symmetric matrices, and $\mathbf{b}_k$ and $\mathbf{A}_k$ are calculated based on (20)-(22).

The problem in (42) can be solved with Algorithm 2. Compared with that of the problem in (39), the complexity of solving (42) is reduced since $\bar{\mathbf{W}}$ in (42) is a real matrix.

V. SIMULATIONS RESULTS

In this section, we provide some numerical results to demonstrate the performances of integrated FD-MIMO radar-communication systems with different parameter configurations. We set $f_0 = 10\text{GHz}$, $\sigma^2 = 10^{-16}$, and the total bandwidth $W = 10\text{MHz}$.

A. The Effects of Different Parameters on Radar/Communication Performance

First, in Figs. 1-3, we demonstrate the effect of the pulse period T on the radar and communication performances when $\Delta f = \frac{1}{T}$. In Fig. 1, we illustrate the relationship between the CRLB and T with varying communication sequences. We assume that $K = 1, A = 1, M = N = 10$ and $M_p = 2$. The position of the target is $(r_1 = 100\text{m}, \theta_1 = \frac{\pi}{20})$. The optimal beamforming vector for minimizing the CRLB is determined by Algorithm 1. Because $M - 1 + L = WT$, for given values of M and W , we can calculate the T s (the x coordinates of Figs. 1-3) and the corresponding minimum CRLBs when $L = 4, 5, \dots, 20$. The line with the “simplified” label is plotted based on (19)-(22). Although $\tau_{mnk} \ll T$ is not satisfied, the CRLB in this simplified case can serve as a lower bound, as shown in Fig. 1. From the analysis results in Sec. III-A, we can see that when $\Delta f = \frac{1}{T}$, the CRLB in

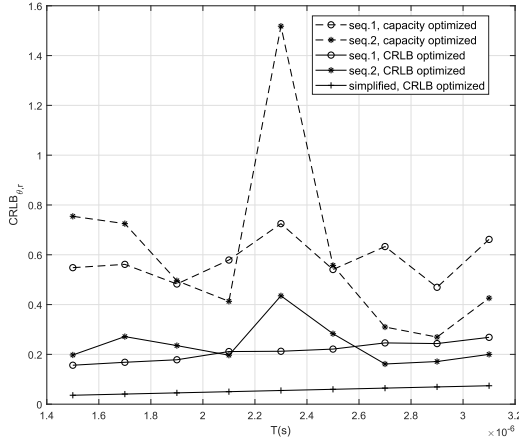


Fig. 2. CRLB vs. T with different communication sequences ($M = N = 12$, $\Delta f = \frac{1}{T}$).

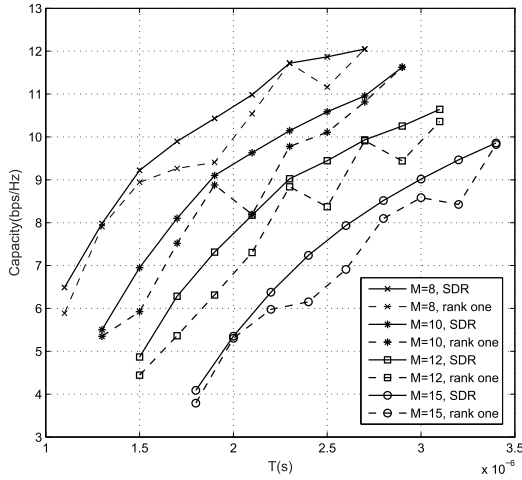


Fig. 3. Spectral efficiency vs. T with different numbers of antennas ($\Delta f = \frac{1}{T}$).

the simplified case varies inversely with T and is independent of the communication sequence. Therefore, in the simplified case, increasing T degrades the estimation performance. The curves with the “CRLB optimized” labels are plotted based on the accurate CRLB matrix in (18). From the analysis in Sec. II-B, we can see that when $\tau_{mnk} \ll T$ is not satisfied, the symbol sequence $\{e^{j\varphi_1}, \dots, e^{j\varphi_L}\}$ influences the CRLB. In Figs. 1 and 2, ‘seq.1’ is $\{0, 0, \dots, 0\}$ and ‘seq.2’ is $\{0, \pi, 0, \pi, \dots\}$. Some communication sequences, such as ‘seq.2’, cause the CRLB to fluctuate with T , especially when T is small. With increasing T , the gap between the curve with ‘simplified’ and those with ‘CRLB optimized’ decreases gradually. In Fig. 2, we set $M = N = 12$, and the other parameters are the same as those in Fig. 1. By comparing Figs. 1 and 2, we can see that the estimation performance benefits from increasing M and N .

In Fig. 3, we show the effect of T on the communication performance $\frac{C}{TW}$ with varying M, N . All other parameter settings are the same as those in Fig. 1. When $\Delta f = \frac{1}{T}$, the maximum capacity is achieved by solving (41) without considering the CRLB constraint. Because $TW = M - 1 + L$, for given

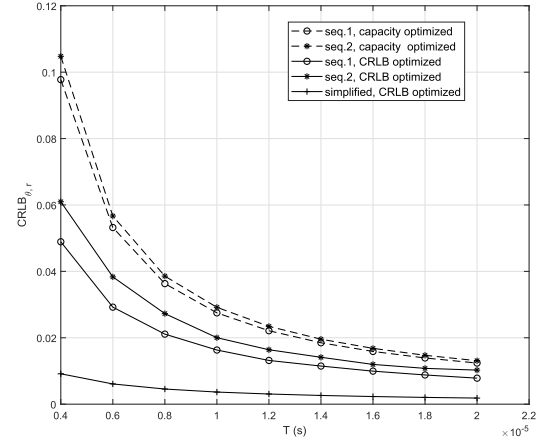


Fig. 4. CRLB vs. T with different communication sequences ($M = N = 10$, $\Delta f = \frac{1}{T}$).

values of T and W , the number of embedded communication symbols L decreases with increasing M . Therefore, although increasing M can lead to an increase in SNR_l , the spectral efficiency still suffers from degradation due to the decreased L , as shown in Fig. 3. Furthermore, we can see that for a given M , the SDR solution of spectral efficiency improves with increasing T . The curves with ‘rank one’ labels correspond to the optimal spectral efficiency with the rank-one solution $\mathbf{w}_{d^*}$. In (41), the rank of the optimal SDR solution $\mathbf{W}_{SDR}^*$ is affected by T . When $\mathbf{W}_{SDR}^*$ is an approximate rank-one matrix, the rank-one solution is close to the SDR solution. To determine the CRLB performance when $\mathbf{w}$ is optimized to maximize the capacity, we calculate the CRLB with the optimal solution to (41) (the curves with ‘capacity optimized’ labels in Figs. 1-2). By comparing the CRLB values of the curves with ‘CRLB optimized’ and ‘capacity optimized’ labels, we can see that even when $K = 1$ (the target is also the communication receiver), the goals of optimizing the radar performance and optimizing the communication performance are not unified. This means that we need to optimize $\mathbf{w}$ to achieve the best tradeoff between the radar and communication performances of the system.

Figs. 4-6 exhibit the effects of the pulse period T on the radar and communication performances when $\Delta f = \frac{1}{T}$. In Fig. 4, we set $M = N = 10$ and in Fig. 5, we set $M = N = 12$. The other parameter settings are the same as those in Fig. 1. Similar to Fig. 1, the curves with ‘CRLB optimized’ labels are plotted according to Algorithm 1, and the curves with ‘capacity optimized’ labels are plotted by calculating the CRLB with $\mathbf{w}$, which is obtained by solving (41). Because $\frac{LM}{T} = W$, for given values of M and W , we can calculate T s and the corresponding minimum CRLBs when $L = 4, 5, \dots, 20$. A comparison between Fig. 4 and Fig. 5 shows that increasing M or N can improve the CRLB. From the analysis results in Sec. III-A, we can see that when $\Delta f = \frac{1}{T_p} = \frac{W}{M}$, the CRLB in the simplified case is proportional to T . Therefore, contrary to the case with $\Delta f = \frac{1}{T}$, increasing T improves the estimation performance when $\Delta f = \frac{1}{T_p}$. Moreover, from the curves with ‘CRLB optimized’ labels,

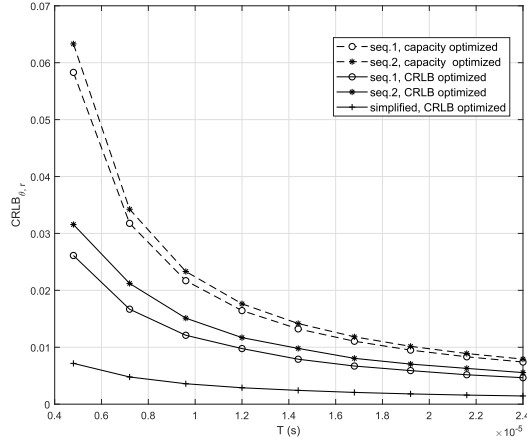


Fig. 5. CRLB vs. T with different communication sequences ($M = N = 12$, $\Delta f = \frac{1}{T_p}$).

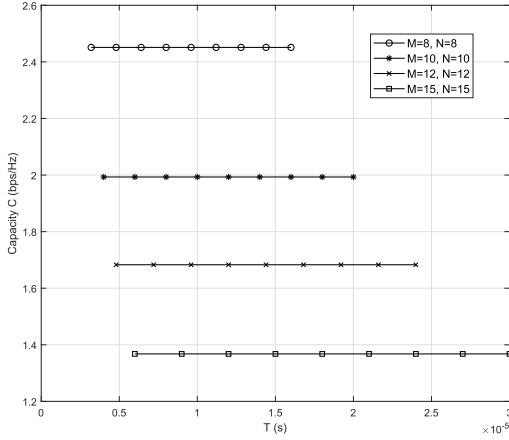


Fig. 6. Spectral efficiency vs. T with different numbers of antennas ($\Delta f = \frac{1}{T_p}$).

we can see that the influence of the communication sequence on the CRLB decreases with increasing T , and an accurate CRLB converges gradually to that in the simplified case. The gap between the CRLB with ‘CRLB optimized’ and that with ‘capacity optimized’ also diminishes with increasing T . This means that when $K = 1$, $\Delta f = \frac{1}{T_p}$ and T is large, the goals of optimizing the radar performance and optimizing the communication performance of the system are almost unified. On the other hand, from Fig. 6, we can see that the spectral efficiency remains unchanged with increasing T and decreases with increasing M . This is because the spectral efficiency $\frac{C}{WT} = \frac{\log(1+A^2|wh|^2/W\sigma^2)}{M}$, which is independent of T and varies inversely with M . Comparing Figs. 1-3 and Figs. 4-6, we can see that for given values of M, W, T , the transmit waveform with $\Delta f = \frac{1}{T_p}$ can achieve better estimation performance than that with $\Delta f = \frac{1}{T}$. On the other hand, when $\Delta f = \frac{1}{T}$, more communication symbols can be embedded in the transmit waveform. Therefore, the waveform with $\Delta f = \frac{1}{T}$ achieves better communication performance than that with $\Delta f = \frac{1}{T_p}$.

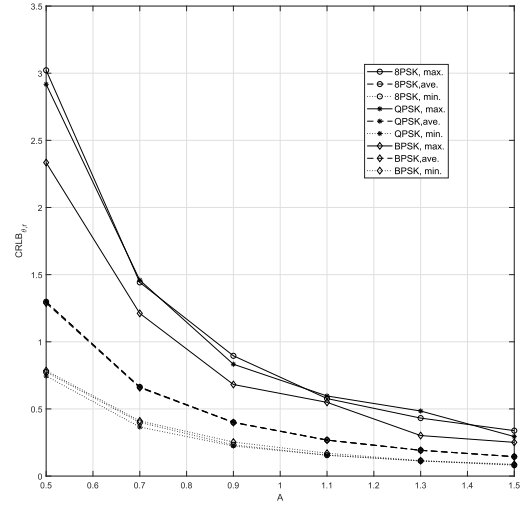


Fig. 7. Average CRLBs with different values of M_p .

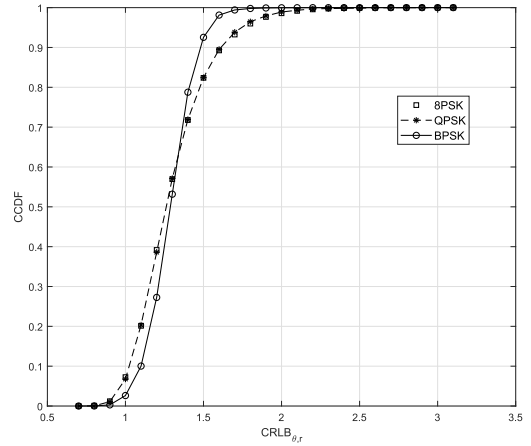
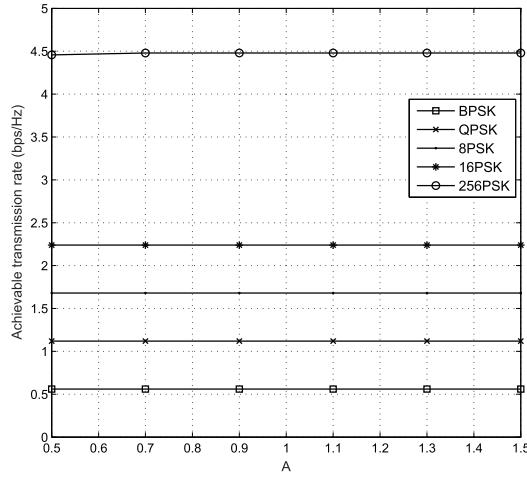
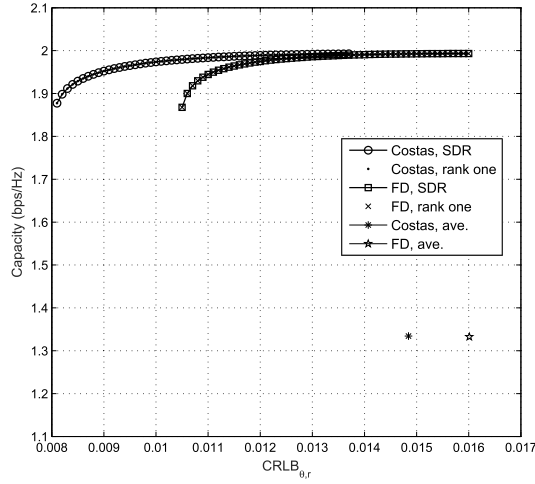


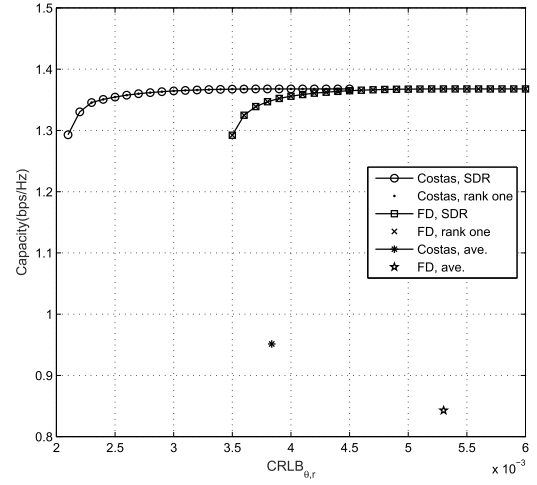
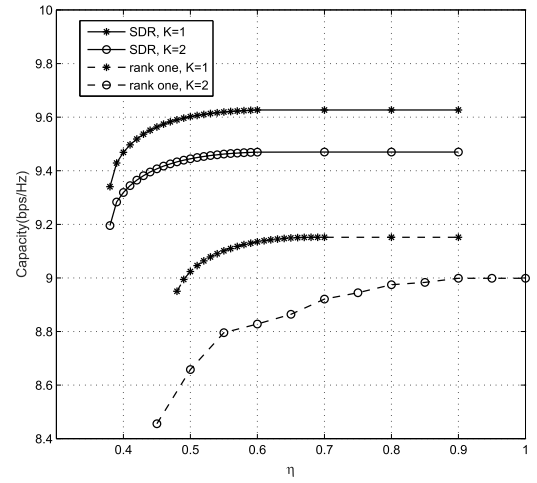
Fig. 8. CCDFs of the CRLBs obtained with different values of M_p .

Since the symbol sequence $\{e^{j\varphi_1}, \dots, e^{j\varphi_L}\}$ influences the CRLB, in Fig. 7, we display the effects of the modulation order and random communication sequence on the radar performance. We set $M_p \in \{2, 4, 8\}$, $L = 14$, $K = 1$ and $M = N = 12$. The average CRLB is obtained by traversing 10^4 communication sequences (curves with ‘ave.’). Furthermore, we select the maximum CRLB and minimum CRLB from the 10^4 samples. The average CRLBs and the minimum CRLBs with different values of M_p are almost the same. Although there exist some communication sequences that result in large CRLBs, especially communication sequences with 8PSK, the number of such sequences is small. As shown in Fig. 8, even in sequences with 8PSK, only approximately 2% sequences have CRLBs greater than 1.6. Removing these sequences improves the CRLB performance with a slight rate loss. On the other hand, from Fig. 9, we can see that the achievable transmission rate (calculated with (27)) increases with increasing M_p . Therefore, from the perspectives of both radar and communication, it is appropriate to design the transmit waveform with high-order modulation and “good” symbol sequences with low CRLBs.


 Fig. 9. Achievable transmission rates with different values of M_p .

 Fig. 10. Tradeoff performance between the CRLB and capacity for $M = N = 10$.

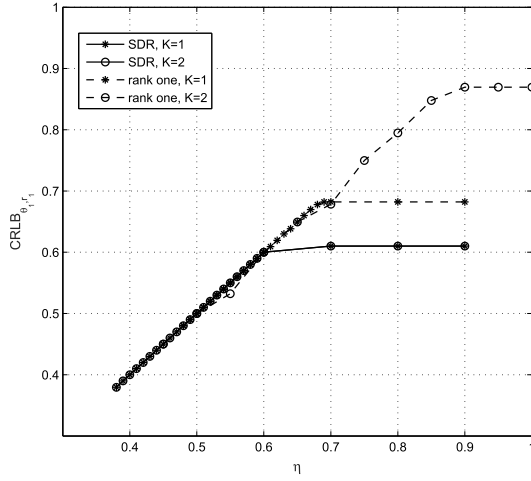
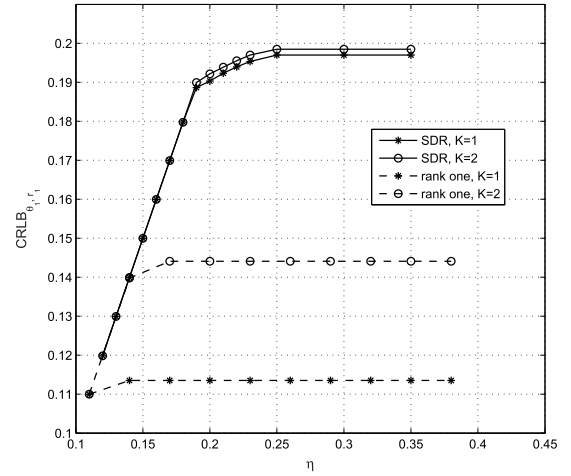
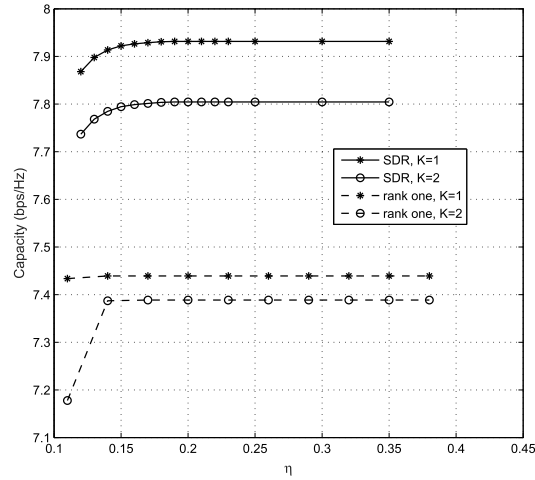
B. Tradeoff Between Radar and Communication Performances

In [20], the Costas frequency coding scheme was used for the FD-MIMO radar-communication system without considering transmit beamforming. In Figs. 10 and 11, we show the best tradeoff between the radar and communication performances of the system by optimizing the transmit beamforming operation in both the Costas frequency coding and FD schemes. In Fig. 10, we set $M = N = 10$, $A = 1$, $K = 1$, $L = 12$, and $\Delta f = \frac{1}{T_p}$, and in Fig. 11, we set $M = N = 15$. The location of the target is ($r = 100m$, $\theta = \pi/20$). For comparison purposes, the capacities and CRLBs of basic Costas frequency coding and FD schemes without transmit beamforming are plotted as a benchmark (the points with ‘Costas, ave.’ and ‘FD, ave.’ labels). In this case, $\mathbf{w} = [\frac{1}{\sqrt{M}}, \frac{1}{\sqrt{M}}, \dots, \frac{1}{\sqrt{M}}]$. To achieve the best tradeoff between the CRLB and capacity, Algorithm 2 is implemented by setting $\eta = 0.008, 0.009, \dots, 0.02$ in Fig. 10 and $\eta = 2 \times 10^{-3}, 2.1 \times 10^{-3}, \dots, 6 \times 10^{-3}$ in Fig. 11. We can obtain the corresponding CRLBs and capacities by utilizing


 Fig. 11. Tradeoff performance between the CRLB and capacity for $M = N = 15$.

 Fig. 12. The optimal capacity vs. the CRLB constraint η ($M = N = 10$).

the optimal solution of Algorithm 2. For these parameter settings, because the SDR solution of algorithm 2 is a rank-one matrix, the optimal solution $\mathbf{W}_r^*$ selected from random samples satisfies $\mathbf{W}_{SDR}^* = \mathbf{W}_r^*$. Therefore, the curves with ‘Costas, rank one.’ or ‘FD, rank one.’ labels overlap with those possessing ‘Costas, SDR’ or ‘FD, SDR’ labels. By comparing the schemes without transmit beamforming, we find that for a given CRLB, the channel capacity can be improved significantly by optimizing the transmit beamforming operation. Moreover, when the CRLB is small, the corresponding channel capacity is close to the maximum value. The results are consistent with those in Fig. 3.

Figs. 12-15 illustrate the tradeoff performance between radar and communication when $\Delta f = \frac{1}{T_p}$ and $K = 1, 2$. When $K = 1$, the location of the target is ($r_1 = 100$, $\theta_1 = \pi/20$), and when $K = 2$, the locations of the two targets are ($r_1 = 100$, $\theta_1 = \pi/20$), ($r_2 = 110$, $\theta_2 = \pi/10$). When $K = 2$, we assume that target 2 is equipped with a communication receiver and that the CRLB of target 1 should be smaller than η . In Figs. 12-15, we set $M_p = 2$, $L = 12$ and $A = 1$.

Fig. 13. The actual CRLB vs. the CRLB constraint η ($M = N = 10$).Fig. 15. The actual CRLB vs. the CRLB constraint η ($M = N = 10$).Fig. 14. The optimal capacity vs. the CRLB constraint η ($M = N = 15$).

In Figs. 12-13, $M = N = 10$ and in Figs. 14-15, $M = N = 15$. For a given η , we execute Algorithm 2 to obtain the optimal beamforming vector $\mathbf{w}^*$, and then the corresponding CRLB of target 1 and capacity of target 2 are calculated with $\mathbf{w}^*$. Because the optimal SDR solution $\mathbf{W}_{SDR}^*$ is not the rank 1 matrix, the capacity and CRLB calculated with the rank-one solution $\mathbf{W}_r^*$ are different from those calculated with $\mathbf{W}_{SDR}^*$. From Fig. 12, it is observed that in the single-target scenario, the channel capacity can reach the maximum value rapidly with the increase of η . This is because in this case, there is no significant conflict between maximizing the transmission rate and minimizing the CRLB. However, when $K = 2$, there is an obvious compromise between the CRLB of target 1 and the transmission rate of target 2. That is, when target 1 lowers the requirement for the CRLB, target 1 can achieve a higher transmission rate. As shown in Figs. 12 and 13 (the curves with ‘ $K = 2$, rank one’ labels), when $\eta = 0.45$, the CRLB of target 1 is 0.45, and the capacity of target 2 is only approximately 8.4 bps/Hz. When $\eta > 0.9$, the CRLB of target 1 is 0.9, and the capacity of target 2 achieves a maximum of 9 bps/Hz. From Figs. 14 and 15, we can see that increasing M is beneficial for

improving the tradeoff performance. In both cases with $K = 1$ and $K = 2$, the maximum channel capacity is achieved when the CRLB is small. For example (see the curves with ‘rank one, $K = 2$ ’ labels), when the CRLB of target 1 is 0.14, the capacity of target 2 reaches 7.4 bps/Hz.

VI. CONCLUSION

In this paper, we propose the design of an integrated radar-communication waveform by embedding phase-modulated communication signals in the transmit waveform of FD-MIMO radar. Transmit beamforming is employed to improve the radar and communication performances of the integrated system. An embedded communication sequence destroys the orthogonality of waveforms on different transmission antennas and further affects the radar estimation performance of the system. To understand the influences of the communication parameters, such as the symbol sequence and modulation order, on radar performance, we first derive the CRLB of multitarget location estimation by considering different transmission delays. On the other hand, we analyze the channel capacity and achievable transmission rate between the dual-function transmitter and the communication receiver. Since transmit beamforming affects both the radar and communication performance of the system, it is optimized by maximizing the transmission rate under the constraint of the CRLB. Since the optimization problem is nonconvex, we combine the SPCA method and SDR to solve the above problem. Simulation results reveal the effects of different parameters on radar/communication performance, and the best tradeoff between the radar and communication performances of the system.

APPENDIX A DERIVATION OF $\mathbf{J}(\mathbf{u})$

The FIM for the unknown parameter vector $\mathbf{u}$ is derived in this Appendix. By ignoring the couplings between the target locations and propagation attenuations [27], the matrix $\mathbf{J}(\gamma)$

can be written as

$$\mathbf{J}(\gamma) = \frac{2}{\sigma_w^2} \text{Re} \begin{bmatrix} \mathbf{A}_1^{(1)} & \mathbf{A}_2^{(1)} & \dots & \mathbf{A}_K^{(1)} & \mathbf{0} \\ \mathbf{A}_1^{(2)} & \mathbf{A}_2^{(2)} & \dots & \mathbf{A}_K^{(2)} & \mathbf{0} \\ \vdots & \vdots & & \vdots & \vdots \\ \mathbf{A}_1^{(K)} & \mathbf{A}_2^{(K)} & \dots & \mathbf{A}_K^{(K)} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{C} \end{bmatrix} \quad (43)$$

where $\mathbf{C}$ is an $MNK \times MNK$ matrix that has no effect on the CRLB of the location estimation. In (43), the main diagonal matrix $\mathbf{A}_k^{(k)}$ is only related to the location of the k th target, and it has a major impact on the CRLB of (θ_k, r_k) . The matrix $\mathbf{A}_k^{(k')}$ on the nonprincipal diagonal denotes the coupling of the k th target and the k' th target in the location estimation. According to (10), $\mathbf{A}_k^{(k')}$ is an $MN * MN$ block diagonal matrix that can be written as

$$\mathbf{A}_k^{(k')} = \begin{bmatrix} \mathbf{A}_k^{1k'} & \dots & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & \dots & \mathbf{A}_k^{Nk'} \end{bmatrix} \quad (44)$$

where $\mathbf{A}_k^{nk'}$ is the following $M \times M$ matrix:

$$\mathbf{A}_k^{nk'} = \begin{bmatrix} w_1^* w_1 a_{k11}^{nk'} & w_1^* w_2 a_{k12}^{nk'} & \dots & w_1^* w_M a_{k1M}^{nk'} \\ w_2^* w_1 a_{k21}^{nk'} & w_2^* w_2 a_{k22}^{nk'} & \dots & w_2^* w_M a_{k2M}^{nk'} \\ \vdots & \vdots & & \vdots \\ w_M^* w_1 a_{kM1}^{nk'} & w_M^* w_2 a_{kM2}^{nk'} & \dots & w_M^* w_M a_{kMM}^{nk'} \end{bmatrix} \quad (45)$$

where

$$\begin{aligned} a_{kmm'}^{nk'} &= \varepsilon_{mnk}^* \varepsilon_{m'nk} \sum_{\tilde{m}=1}^M \exp(j2\pi\Delta f(c(m)\tau_{mnk'} - c(m')\tau_{m'nk})) \\ &\quad \times \{\lambda^*(\tau_{mnk'}, m, \tilde{m})(j2\pi c(m)\Delta f) + \dot{\lambda}^*(\tau_{mnk'}, m, \tilde{m})\} \\ &\quad \times \{(-j2\pi c(m')\Delta f)\lambda(\tau_{m'nk}, m', \tilde{m}) + \dot{\lambda}(\tau_{m'nk}, m', \tilde{m})\}. \end{aligned} \quad (46)$$

In (46), $\lambda(\tau_{mnk}, m, \tilde{m})$ is given in (6) and $\dot{\lambda}(\tau_{mnk}, m, \tilde{m}) = \frac{\partial \lambda(\cdot)}{\partial \tau_{mnk}}$. For the convenience of later calculations, the m' th column of $\mathbf{A}_k^{nk'}$ is denoted as

$$[\mathbf{A}_k^{nk'}](:, m') = \mathbf{\Lambda}_{km'}^{nk'} \mathbf{w}_{m'}^H \quad (47)$$

where $\mathbf{\Lambda}_{km'}^{nk'}$ is a diagonal matrix with diagonal elements $[\mathbf{\Lambda}_{km'}^{nk'}]_{m,m} = a_{kmm'}^{nk'}$.

Let $\mathbf{Q}_k = \begin{bmatrix} \mathbf{x}_k \\ \mathbf{z}_k \end{bmatrix}$ in (9). Based on (8), (9) and (43), we can obtain the following expression for $\mathbf{J}(\mathbf{u})$:

$$\begin{aligned} \mathbf{J}(\mathbf{u}) &= \frac{2}{\sigma_w^2} \text{Re} \begin{bmatrix} \mathbf{Q}_1 \mathbf{A}_1^{(1)} \mathbf{Q}_1^H & \mathbf{Q}_1 \mathbf{A}_2^{(1)} \mathbf{Q}_2^H & \dots & \mathbf{Q}_1 \mathbf{A}_K^{(1)} \mathbf{Q}_K^H & \mathbf{0} \\ \mathbf{Q}_2 \mathbf{A}_1^{(2)} \mathbf{Q}_1^H & \mathbf{Q}_2 \mathbf{A}_2^{(2)} \mathbf{Q}_2^H & \dots & \mathbf{Q}_2 \mathbf{A}_K^{(2)} \mathbf{Q}_K^H & \mathbf{0} \\ \vdots & \vdots & & \vdots & \vdots \\ \mathbf{Q}_K \mathbf{A}_1^{(K)} \mathbf{Q}_1^H & \mathbf{Q}_K \mathbf{A}_2^{(K)} \mathbf{Q}_2^H & \dots & \mathbf{Q}_K \mathbf{A}_K^{(K)} \mathbf{Q}_K^H & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{C} \end{bmatrix} \end{aligned}$$

For simplicity, we denote $\mathbf{G}_k^{(k')} = \mathbf{Q}_{k'} \mathbf{A}_k^{(k')} \mathbf{Q}_k^H$. Based on (44) and (47), we obtain

$$\begin{aligned} [\mathbf{G}_k^{(k')}]_{1,1} &= \sum_{n=1}^N \sum_{m=1}^M [\mathbf{x}_{k'}]((n-1)M+1:nM) [\mathbf{x}_k]_{(n-1)M+m} \\ &\quad \times \mathbf{\Lambda}_{km}^{nk'} w_m \mathbf{w}^H \\ &= \mathbf{w} \mathbf{G}_a^{kk'} \mathbf{w}^H \end{aligned} \quad (48)$$

$$\begin{aligned} [\mathbf{G}_k^{(k')}]_{1,2} &= \sum_{n=1}^N \sum_{m=1}^M [\mathbf{x}_{k'}]((n-1)M+1:nM) \frac{2}{C} \mathbf{\Lambda}_{km}^{nk'} w_m \mathbf{w}^H \\ &= \mathbf{w} \mathbf{G}_b^{kk'} \mathbf{w}^H \end{aligned} \quad (49)$$

$$\begin{aligned} [\mathbf{G}_k^{(k')}]_{2,1} &= \sum_{n=1}^N \sum_{m=1}^M [\mathbf{z}_{k'}]((n-1)M+1:nM) [\mathbf{x}_k]_{(n-1)M+m} \\ &\quad \times \mathbf{\Lambda}_{km}^{nk'} w_m \mathbf{w}^H \\ &= \mathbf{w} \mathbf{G}_c^{kk'} \mathbf{w}^H \end{aligned} \quad (50)$$

$$\begin{aligned} [\mathbf{G}_k^{(k')}]_{2,2} &= \sum_{n=1}^N \sum_{m=1}^M [\mathbf{z}_{k'}]((n-1)M+1:nM) \frac{2}{C} \mathbf{\Lambda}_{km}^{nk'} w_m \mathbf{w}^H \\ &= \mathbf{w} \mathbf{G}_d^{kk'} \mathbf{w}^H \end{aligned} \quad (51)$$

where

$$[\mathbf{G}_a^{kk'}](m,:) = \sum_{n=1}^N [\mathbf{x}_{k'}]((n-1)M+1:nM) [\mathbf{x}_k]_{(n-1)M+m} \mathbf{\Lambda}_{km}^{nk'}, \quad (52)$$

$$[\mathbf{G}_b^{kk'}](m,:) = \sum_{n=1}^N [\mathbf{x}_{k'}]((n-1)M+1:nM) \frac{2}{C} \mathbf{\Lambda}_{km}^{nk'} \quad (53)$$

$$[\mathbf{G}_c^{kk'}](m,:) = \sum_{n=1}^N [\mathbf{z}_{k'}]((n-1)M+1:nM) [\mathbf{x}_k]_{(n-1)M+m} \mathbf{\Lambda}_{km}^{nk'}, \quad (54)$$

and

$$[\mathbf{G}_d^{kk'}](m,:) = \sum_{n=1}^N \{[\mathbf{z}_{k'}]((n-1)M+1:nM) \frac{2}{C} \mathbf{\Lambda}_{km}^{nk'}\}. \quad (55)$$

REFERENCES

- [1] C. Sturm and W. Wiesbeck, "Waveform design and signal processing aspects for fusion of wireless communications and radar sensing," *Proc. IEEE*, vol. 99, no. 7, pp. 1236–1259, Jul. 2011.
- [2] S. Quan, W. Qian, J. Guq, and V. Zhang, "Radar-communication integration: An overview," in *Proc. IEEE Int. Conf. Adv. Infocomm Technol.*, Nov. 2014, pp. 98–103.
- [3] K. V. Mishra, M. R. B. Shankar, V. Koivunen, B. Ottersten, and S. A. Vorobyov, "Toward millimeter-wave joint radar communications," *IEEE Signal Process. Mag.*, vol. 36, no. 5, pp. 100–114, Sep. 2019.
- [4] G. Hua and S. S. Abeysekera, "Collocated MIMO radar transmit beamforming using orthogonal waveforms," in *Proc. IEEE Int. Conf. Acoust., Speech Signal Process. (ICASSP)*, Mar. 2012, pp. 2453–2456.
- [5] D. R. Fuhrmann and G. S. Antonio, "Transmit beamforming for MIMO radar systems using partial signal correlation," in *Proc. Conf. Rec. 38th Asilomar Conf. Signals, Syst. Comput.*, Nov. 2004, pp. 295–299.
- [6] A. Hassanien, M. G. Amin, Y. D. Zhang, and F. Ahmad, "Phase-modulation based dual-function radar-communications," *IET Radar, Sonar Navigat.*, vol. 10, no. 8, pp. 1411–1421, Oct. 2016.
- [7] A. Hassanien, M. G. Amin, Y. D. Zhang, and F. Ahmad, "Dual-function radar-communications: Information embedding using sidelobe control and waveform diversity," *IEEE Trans. Signal Process.*, vol. 64, no. 8, pp. 2168–2181, Apr. 2016.

- [8] A. Hassanien, M. G. Amin, Y. D. Zhang, and B. Himed, "A dual-function MIMO radar-communications system using PSK modulation," in *Proc. 24th Eur. Signal Process. Conf. (EUSIPCO)*, Aug. 2016, pp. 1613–1617.
- [9] I. P. Eedara, A. Hassanien, M. G. Amin, and B. D. Rigling, "Ambiguity function analysis for dual-function radar communications using PSK signaling," in *Proc. 52nd Asilomar Conf. Signals, Syst., Comput.*, Oct. 2018, pp. 900–904.
- [10] E. BouDaher, A. Hassanien, E. Aboutanos, and M. G. Amin, "Towards a dual-function MIMO radar-communication system," in *Proc. IEEE Radar Conf. (RadarConf)*, May 2016, pp. 1–6.
- [11] F. Liu, C. Masouros, A. Li, H. Sun, and L. Hanzo, "MU-MIMO communications with MIMO radar: From co-existence to joint transmission," *IEEE Trans. Wireless Commun.*, vol. 17, no. 4, pp. 2755–2770, Apr. 2018.
- [12] F. Liu, L. Zhou, C. Masouros, A. Li, W. Luo, and A. Petropulu, "Towards dual-functional radar-communication systems: Optimal waveform design," *IEEE Trans. Signal Process.*, vol. 66, no. 16, pp. 4264–4279, Jun. 2018.
- [13] S. H. Dokhanchi, B. S. Mysore, K. V. Mishra, and B. Ottersten, "A mmWave automotive joint radar-communications system," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 55, no. 3, pp. 1241–1260, Jun. 2019.
- [14] P. Kumari, S. A. Vorobyov, and R. W. Heath, Jr., "Adaptive virtual waveform design for millimeter-wave joint communication–radar," *IEEE Trans. Signal Process.*, vol. 68, pp. 715–730, Nov. 2020.
- [15] P. Kumari, D. H. N. Nguyen, and R. W. Heath, "Performance trade-off in an adaptive IEEE 802.11AD waveform design for a joint automotive radar and communication system," in *Proc. IEEE Int. Conf. Acoust., Speech Signal Process. (ICASSP)*, Mar. 2017, pp. 4281–4285.
- [16] J. F. Li, X. F. Zhang, and D. F. Jiang, "DOD and DOA estimation for bistatic coprime MIMO radar based on combined ESPRIT," in *Proc. CIE Int. Conf. Radar (RADAR)*, Oct. 2017, pp. 1–4.
- [17] W.-Q. Wang, "Frequency diverse array antenna: New opportunities," *IEEE Antennas Propag. Mag.*, vol. 57, no. 2, pp. 145–152, Apr. 2015.
- [18] P. Antonik, M. C. Wicks, H. D. Griffiths, and C. J. Baker, "Frequency diverse array radars," in *Proc. IEEE Conf. Radar*, Apr. 2006, pp. 215–217.
- [19] J. Xiong, W.-Q. Wang, and K. Gao, "FDA-MIMO radar range–angle estimation: CRLB, MSE, and resolution analysis," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 54, no. 1, pp. 284–294, Feb. 2018.
- [20] S. Y. Nusenu and W.-Q. Wang, "Dual-function FDA MIMO radar-communications system employing costas signal waveforms," in *Proc. IEEE Radar Conf. (RadarConf)*, Apr. 2018, pp. 33–38.
- [21] S. Y. Nusenu, W.-Q. Wang, and A. Basit, "Time-modulated FD-MIMO array for integrated radar and communication systems," *IEEE Antennas Wireless Propag. Lett.*, vol. 17, no. 6, pp. 1015–1019, Jun. 2018.
- [22] S. M. Kay, *Fundamentals of Statistical Signal Processing-Estimation Theory*. Upper Saddle River, NJ, USA: Prentice-Hall, 1993.
- [23] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.
- [24] A. Beck, A. Ben-Tal, and L. Tetruashvili, "A sequential parametric convex approximation method with applications to nonconvex truss topology design problems," *J. Global Optim.*, vol. 47, no. 1, pp. 29–51, May 2010.
- [25] D. P. Bertsekas, *Nonlinear Programming*. Belmont, MA, USA: Athena Scientific, 1995.
- [26] Z.-Q. Luo, W.-K. Ma, A. So, Y. Ye, and S. Zhang, "Semidefinite relaxation of quadratic optimization problems," *IEEE Signal Process. Mag.*, vol. 27, no. 3, pp. 20–34, May 2010.
- [27] H. Godrich, A. P. Petropulu, and H. V. Poor, "Power allocation strategies for target localization in distributed multiple-radar architectures," *IEEE Trans. Signal Process.*, vol. 59, no. 7, pp. 3226–3240, Jul. 2011.



Lan Tang (Member, IEEE) received the Ph.D. degree in communication and information science from Southeast University, Nanjing, China, in 2009. Since 2009, she has been with the School of Electronic Science and Engineering, Nanjing University, China, where she is currently an Associate Professor. In 2014, she was a Visiting Researcher with Columbia University, USA. Her current research interests include optimization and machine learning in wireless communications, and statistical signal processing applied for radar and communications.



Yechao Bai received the B.S. and Ph.D. degrees in electronic science and engineering from Nanjing University, Nanjing, China, in 2005 and 2010, respectively.

Since 2010, he has been with Nanjing University, where he is currently an Associate Professor with the School of Electronic Science and Engineering. His current research interests include array signal processing, target detection, and parameter estimation.



Ying-Chang Liang (Fellow, IEEE) was a Professor with The University of Sydney, Australia, a Principal Scientist and a Technical Advisor with the Institute for Infocomm Research, Singapore, and a Visiting Scholar with Stanford University, USA. He is currently a Professor with the University of Electronic Science and Technology of China, China, where he leads the Center for Intelligent Networking and Communications. He is also a Deputy Director of the Artificial Intelligence Research Institute. His research interests include wireless networking and communications, cognitive radio, symbiotic networks, dynamic spectrum access, the Internet-of-Things, artificial intelligence, and machine learning techniques.

Dr. Liang is a Foreign Member of Academia Europaea. He has been recognized by the Thomson Reuters (Clarivate Analytics) as a Highly Cited Researcher, since 2014. He received the Prestigious Engineering Achievement Award from the Institution of Engineers, Singapore, in 2007, the Outstanding Contribution Appreciation Award from the IEEE Standards Association, in 2011, and the Recognition Award from the IEEE Communications Society Technical Committee on Cognitive Networks, in 2018. He was a recipient of numerous paper awards, including the IEEE Jack Neubauer Memorial Award, in 2014, and the IEEE Communications Society APB Outstanding Paper Award, in 2012. He was the Chair of the IEEE Communications Society Technical Committee on Cognitive Networks. He served as the TPC Chair and the Executive Co-Chair for the IEEE Globecom 2017. He was an Associate Editor-in-Chief of the *Random Matrices: Theory and Applications* (World Scientific). He serves as an Associate Editor-in-Chief for *China Communications*. He is the Founding Editor-in-Chief of the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS: COGNITIVE RADIO SERIES and the Key Founder. He serves as the Editor-in-Chief for the IEEE TRANSACTIONS ON COGNITIVE COMMUNICATIONS AND NETWORKING. He was a Distinguished Lecturer of the IEEE Communications Society and the IEEE Vehicular Technology Society.



Xufeng Zhou received the B.S. degree in mathematics and applied mathematics from Northwest University, Xi'an, China, in 2019. He is currently pursuing the master's degree with the School of Electronic Science and Engineering, Nanjing University, China. His current research interests include waveform design for integrated radar-communication systems and optimization theory in wireless communication.