

Wireless Information and Energy Transfer in Fading Relay Channels

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Abstract—Wireless energy transfer is a promising solution to provide convenient and steady energy supplies for low-power relays. This paper investigates the simultaneous information and energy transfer in fading relay channels, where the relay has no fixed energy supply and replenishes energy from radio frequency signals transmitted by the source. Assume that the relay can switch among energy harvesting, information decoding, and information retransmission in each channel fading state. Our objective is to maximize the ergodic throughput by optimizing the mode switching rule and transmit power jointly under the data and energy causality constraints. When the source knows channel state information (CSI) of all links, to make the problem tractable, for the relay, we neglect the causality constraints during the transmission, and only consider the total data and energy constraints. We thus obtain an upper bound on the ergodic throughput by solving a convex optimization problem. Numerical results show that the achievable rate is very close to the upper bound when we apply the optimized parameters to a practical system. When the source only knows CSI of partial links, the whole transmission process is divided into two phases: the source transmits in the first phase and the relay decodes and forwards received bits using the harvested energy in the second phase. The throughput maximization problem is solved by combining convex optimization, fractional programming, and linear search. We also consider the simplified network topology when a direct link between the source and destination is unavailable. In this network, we propose algorithms based on bisection method to obtain the optimal parameters in information/energy transfer scheduling and power control when the source knows full or partial CSI. The simulation results reveal that the throughput gain brought by wireless powered relaying in different system configurations when the source knows full or partial CSI. Moreover, the effect of the relay position is discussed.

Index Terms—Wireless information and energy transfer, resource allocation, relay, fading channels.

I. INTRODUCTION

ENERGY harvesting is a promising technique to prolong the lifetime of an energy-constrained wireless network.

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In future wireless networks, nodes are envisioned to be capable of harvesting energy from the environment. In addition to the renewable energy resources such as solar and wind, radio signals can be a viable new source due to the emerging wireless energy transfer (WET) technology [1]. The radio frequency (RF)-based energy harvesting is typically suitable for some short-distance and low-power applications. It has been reported that 3.5mW and 1 μ W RF power can be harvested from over distances of 0.6 and 11 meters respectively using Powercast energy harvester [2].

A. Background and Previous Work

Traditional energy harvesting methods based on solar, piezo, and thermal, share a common limitation of being dependent on ambient sources generally beyond control. WET based on RF signals overcomes this lack of control because power can be replenished when desired. On the other hand, since RF signals carry information as well as energy, simultaneous wireless information and power transfer (SWIPT) has attracted significant attention. Recent research revealed the non-trivial tradeoff between information capacity and energy transfer efficiency in different SWIPT systems. Reference [3] analyzed the optimal tradeoff between the achievable rate and the transferred power in a noisy coupled-inductor circuit. In [4], a unified study on SWIPT in the MIMO broadcast channel was performed. The information theoretical studies on SWIPT reveal the fundamental trade-off between information and power transfer by assuming no loss in power on decoding information [3]–[5].

However, due to the practical circuit constraints, the receiver is unable to decode information and harvest energy from the same signal. To achieve SWIPT, some different types of receivers, such as time switching (TS) [4], power switching (PS) [4], and antenna switching (AS) [6] were proposed to split the received signal in two parts, one for energy harvesting (EH) and the other for information decoding (ID). In particular, TS receiver is practically appealing due to the simple hardware implementation. In the multiuser case, when one TS receiver receives information from the transmitter, other TS receivers can harvest energy opportunistically from the same signal. As an initial study on SWIPT in multiuser environment, [4] and [7] investigated the design of the transmit signals in the simplified scenarios with one ID receiver and one EH receiver. [8] extended the result to the MISO beamforming system with multiple ID and EH users.

Recently, some researchers have applied WET to relay networks [9]–[18]. References [9] and [10] assumed both

source and relay have fixed power supply and the destination has EH capability. Their goals were to achieve different tradeoffs for SWIPT by optimizing system parameters, such as transmit power, PS factor, precoders and so on. References [11]–[13] focused on the outage probability and the throughput performance analysis of EH AF (or DF) relaying when TS or PS relaying protocol was adopted. Reference [14] studied the outage probability and ergodic capacity of a dual-hop relaying system where a multi-antenna relay was powered by RF signals. In [15] and [16], the relay switched between EH and information transmission based on the level of the harvested energy. Reference [15] proposed a greedy switching policy where the relay transmitted when its residual energy ensured decoding at the destination. Reference [16] derived analytical expressions of the achievable throughput for the proposed protocol in which the relay continued to harvest energy from RF signals until it had enough energy to transmit. The information/energy cooperation transmission in OFDM relay networks was studied in [17] and [18]. The works above didn't consider the joint mode switching and power control in time-varying channels when the relay is wireless powered by RF signals from the source.

B. Motivation

So far, [22] has studied the power allocation to maximize the ergodic capacity for wireless relay channels when the relay has fixed energy supply. Reference [19] investigated when the receiver should switch between ID mode and EH mode based on the instantaneous CSI in point-to-point wireless link. Motivated by existing work, we consider three-node wireless powered relay systems in a Rayleigh-fading environment. The direct link between the source and destination suffers from severe fading due to the shielding effect so that the source asks a wireless powered relay to assist its information transmission. In such a scenario, we want to address: how shall a relay switch among EH, ID, and information retransmission (IRT) and how to control the transmit power of source and relay based on instantaneous CSI so that the ergodic throughput is maximized under causality constraints.

C. Contribution

In most existing works which adopted TS relaying protocol, the fading channel gains were assumed to be constant over a block time and the block time is divided into fixed three parts: EH, ID and IRT. However, a long codeword may experience multiple channel fading states. To exploit the wireless channel at its best, in this paper, the switching among the EH, ID and IRT depends on time-varying CSI. We design two transmission protocols when the source knows full or partial CSI. The contributions of this work are summarized as follows.

Firstly, we consider the relay system with a severe fading direct link between the source and destination and treat both the full and partial CSI cases: (1) Assume the source knows CSI of all links and can choose direct transmission, relaying transmission or idle mode based on known CSI. Correspondingly, the relay will switch among EH, ID, and IRT in each fading state. We address the throughput maximization problem by optimizing the mode switching rule and transmit

power jointly under the data and energy causality constraints. We obtain an upper bound on the ergodic throughput by neglecting the causality constraints at the relay during the transmission. Numerical results show that the achievable rate is very close to the upper bound when we apply the optimized parameters to a practical system. (2) When the source only knows the CSI of source-relay (S-R) and source-destination (S-D) links, the transmission process consists of two phases: source transmits to the relay or destination in the first phase and relay forwards information signals in the second phase. In this scheme, we address the problem of maximizing the average throughput by varying the transmit power (of source and relay) and transmission mode of the source. By exploiting fractional programming and the Lagrange dual method, an iterative resource allocation algorithm is developed to solve the non-convex problem.

Then, a simplified network topology without direct link is considered again for both full and partial CSI cases. In this network, the source needs to transmit energy signals to the relay. (1) When the source knows the CSI of all links, it can transmit energy (or information) signals to the relay or keep idle. The average throughput is maximized by optimizing the mode switching rule and transmit power jointly. The optimal parameters in the optimal solutions are found by a algorithm based on bisection method. (2) Suppose that the source only knows the CSI of S-D link. The transmission process consists of two phases: the source first transmits information (or energy) signals to the relay, and then the relay forwards information signals to the destination using the harvested energy. We maximize the average throughput through joint power control and scheduling information/energy transfer. The problem is formulated as a non-convex problem and solved by combining convex optimization and linear search.

Finally, numerical results show that the weaker the direct link is or the lower the average transmit power of source is, the higher the throughput gain brought by wireless powered relay is. That means deploying EH relay node is an energy efficient solution to enable the communication between source and destination when there is deep-fading (or no) direct link between the source and destination. Moreover, the throughput achieved by the transmission scheme with partial CSI approaches that with full CSI at source with the strengthening of the direct link. For both full and partial CSI cases, the throughput performance is relatively worse when the relay is placed in the middle of the S-D link. Specially, in partial CSI case, the influence of the relay position on the throughput performance increase as the direct link gets worse. Compared with partial CSI case, in full CSI case, the relay position has a greater effect on the throughput performance.

The remainder of this paper is organized as follows: Section II presents the system model. Section III derives the resource allocation solutions for the relay system with a weak direct link when the source knows full or partial CSI. Section IV studies power control and information/energy scheduling in relay systems without a direct link when the source knows full or partial CSI. Section V presents some numerical results to evaluate the system performance. Finally, Section VI concludes the paper.

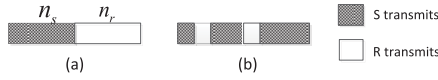


Fig. 1. Transmission protocols (a) partial CSI (b) full CSI.

II. SYSTEM MODEL

Consider a three-node relay channel. The relay node operates in half duplex mode and assists in the communication between the source and the destination by decoding, re-encoding and forwarding the signals from the source. We consider an orthogonal channel model, where the source and relay transmit in different time slots [21]. We assume the small-scale fading of all links in the system follows the Rayleigh distribution. Specially, the S-D link suffers from deep fading due to the shielding effect caused by some physical obstacles. The small-scale fading of S-R, R-D, S-D links are denoted as $h_{sr}, h_{rd}, \sqrt{\alpha}h_{sd}$, where $0 \leq \alpha < 1$ denotes the deep fading coefficient and $\{h_{sr}, h_{rd}, h_{sd}\} \sim \mathcal{CN}(0, 1)$. The transmission distances in the S-R link, S-D link and R-D link are d_{sr}, d_{sd}, d_{rd} , respectively. The path loss in each link is modeled as a power falloff proportional to the squared distance. Hence, the channel power gains in the S-R, S-D, and R-D links ($\gamma_{sr}, \gamma_{sd}, \gamma_{rd}$) are modeled as exponentially distributed random variables with parameters L_{sr}, L_{sd}, L_{rd} . Throughout the paper, we assume that the input codewords are obtained from a random Gaussian codebook. All channels are fast fading so that the total communication time spans many channel coherent periods.

The relay node is assumed to have no fixed power supply and can only harvest energy from RF signals from the source. Due to practical circuit constraints, the relay node cannot decode the information and harvest energy simultaneously, so the TS-based receiver architecture is adopted. When the source transmits information signals to the destination, the relay opportunistically harvests energy from the same signal. We also consider the case when there is no S-D link and thus the source will spend dedicated time to transfer wireless energy to the relay.

We assume that the source knows instantaneous CSI γ_{sr}, γ_{sd} and the relay knows γ_{rd} by exploiting the reciprocity of TDD channel. The relay also knows γ_{sr} by accurate channel estimation. The relay can feedback γ_{rd} to the source via a low-rate feedback channel. Whether the source knows γ_{rd} will affect the cooperative transmission between the source and relay. In this paper, partial CSI means that the source only knows γ_{sr}, γ_{sd} , and full CSI means that the source knows $\gamma_{sr}, \gamma_{sd}, \gamma_{rd}$. The transmission protocols in two cases are shown in Fig. 1. When the source has full CSI, the source can select direct transmission, relaying transmission or idle mode based on known CSI. When the source transmits to the destination directly, the relay is in the EH mode. When the source transmit to the relay, the relay is in the ID mode. When the source has partial CSI, the system cannot choose the transmit node in each fading state so that the transmission process is divided into two phases. Assume the total transmission time is T , and the ratio of the duration

of the first phase to that of the second phase is $\frac{n_s}{n_r}$ which is determined by statistical CSI, as stated in Sec. III. In the first phase ($\frac{T}{1+n_r/n_s}$), the source adapts the transmit power and switches the transmit mode based on γ_{sr}, γ_{sd} . In the second phase ($\frac{T}{1+n_s/n_r}$), the relay adjusts the transmit power according to harvested energy and instantaneous CSI γ_{rd} .

III. INFORMATION AND ENERGY TRANSFER IN SELECTION RELAYING

A. Problem Formulations

In this section, we investigate the optimal resource allocation schemes in selection relaying when the source has full or partial CSI. There is a weak direct link between the source and the destination, i.e., $0 < \alpha < 1$.

In the three-point relay system, in which the relay decodes and re-encodes the packet, the achievable rate is limited by the rate in S-R link. If $\gamma_{sr} < \gamma_{sd}$, the achievable rate of DF relaying is even smaller than that of the direct transmission between source and destination [22]. Therefore, in fading channels, the source will select the DF relaying or direct transmission adaptively according to CSI. The ergodic capacity and corresponding power allocation have been investigated in [22] when the relay has fixed power supply. However, when the relay decodes, re-encodes and retransmits information bits using the harvested energy from the source, how could the system get the most benefit from the relay transmission? Intuitively, the weaker the direct link is and the more energy the relay gets, the greater the throughput improvement resulting from relay transmission is.

Full CSI: Denote the channel gain vector as $\boldsymbol{\gamma} = \{\gamma_{sr}, \gamma_{sd}, \gamma_{rd}\}$. We first consider the problem under a finite-horizon setup. Let $\{\boldsymbol{\gamma}(n)\}_{n=1}^N$ be a specific realization of the fading process. In each time slot, we define two indicator variables $\tilde{\rho}(n), \rho(n)$ to distinguish different transmission modes. Specifically, at time n , if the source transmits, the indicator variable $\tilde{\rho}(n) = 1$: if it transmits information to the relay, the transmit power is $P_{S1}(n)$, the indicator variable $\rho(n) = 1$, and the rate $r_{sr}(n) = \rho(n) \log(1 + \frac{\gamma_{sr}(n)P_{S1}(n)}{N_0})$; and if it transmits to the destination, the transmit power is $P_{S2}(n)$, $\rho(n) = 0$, and the rate $r_{sd}(n) = (\tilde{\rho}(n) - \rho(n)) \log(1 + \frac{\gamma_{sd}(n)P_{S2}(n)}{N_0})$. If the relay transmits the re-encoded signals at time n , the transmit power is $P_R(n)$, $\tilde{\rho}(n) = 0$, and the rate $r_r(n) = (1 - \tilde{\rho}(n)) \log(1 + \frac{\gamma_{rd}(n)P_R(n)}{N_0})$. Moreover, at time n , the transmit power at the source, and the received energy at the relay are given respectively by $p(n) = \rho(n)P_{S1}(n) + (\tilde{\rho}(n) - \rho(n))P_{S2}(n)$ and $g(n) = (\tilde{\rho}(n) - \rho(n))\eta\gamma_{sr}(n)P_{S2}(n)$ where $0 < \eta \leq 1$ denotes the energy conversion efficiency [11]. We impose two types of power constraints to the source: average power constraint and peak power constraint, i.e.,

$$\frac{1}{N} \sum_{n=1}^N p(n) \leq P, \quad 0 \leq P_{S1}(n), \quad P_{S2}(n) \leq P_p \quad (1)$$

where P and P_p denote the average power constraint and peak power constraint of the source, respectively. Furthermore, for

the relay, at any time, both the data and the energy should meet the causality constraints, i.e.,

$$\sum_{n=1}^m r_r(n) \leq \sum_{n=1}^m r_{sr}(n),$$

$$\sum_{n=1}^m (1 - \tilde{\rho}(n)) P_R(n) \leq \sum_{n=1}^m g(n), \quad m = 1, \dots, N. \quad (2)$$

According to the max-flow-min-cut theorem, the ergodic capacity bound is given by

$$NR \leq \min \left\{ \sum_{n=1}^N R_1(n), \sum_{n=1}^N R_2(n) \right\} \quad (3)$$

where $R_1(n) = r_{sr}(n) + r_{sd}(n)$, $R_2(n) = r_{s1}(n) + r_{sd}(n) + r_r(n)$, $r_{s1}(n) = \rho(n) \log(1 + \frac{\gamma_{sd}(n) P_{S1}(n)}{N_0})$. The total transmission bits from the source should equal the total received bits by the destination. Hence, $\sum_{n=1}^N r_r(n) = \sum_{n=1}^N (r_{sr}(n) - r_{s1}(n))$. Then we can formulate the following optimization problem

$$\begin{aligned} & \max_{\{P_{S1}(n), P_{S2}(n), \rho(n), \tilde{\rho}(n), P_R(n)\}} R, \\ & \text{s.t. (1), (2), (3),} \\ & \sum_{n=1}^N r_r(n) = \sum_{n=1}^N (r_{sr}(n) - r_{s1}(n)), \\ & \{\rho(n), \tilde{\rho}(n)\} \in \{0, 1\}. \end{aligned} \quad (4)$$

The above optimization problem is a mixed integer program and is hard to solve. More importantly, it needs to know the channel realizations $\{\mathbf{y}(n), n = 1, \dots, N\}$ in advance which is impractical. We therefore consider another formulation in the asymptotic regime of $N \rightarrow \infty$, where the variables are functions of the instantaneous channels only, i.e., $P_{S1}(\mathbf{y}), P_{S2}(\mathbf{y}), \rho(\mathbf{y}), \tilde{\rho}(\mathbf{y}), P_R(\mathbf{y})$. We first relax $\{\rho(\mathbf{y}), \tilde{\rho}(\mathbf{y})\} \in \{0, 1\}$ to $\{\rho(\mathbf{y}), \tilde{\rho}(\mathbf{y})\} \in [0, 1]$ and it will turn out that the optimal solutions to the relaxed problem are still binary. The corresponding rates, transmit power and received energy also become functions of the instantaneous channels, i.e., $r_{sr}(\mathbf{y}) = \rho(\mathbf{y}) \log(1 + \frac{\gamma_{sr} P_{S1}(\mathbf{y})}{N_0})$, $r_{sd}(\mathbf{y}) = (\tilde{\rho}(\mathbf{y}) - \rho(\mathbf{y})) \log(1 + \frac{\gamma_{sd} P_{S2}(\mathbf{y})}{N_0})$, $r_{s1}(\mathbf{y}) = \rho(\mathbf{y}) \log(1 + \frac{\gamma_{sd} P_{S1}(\mathbf{y})}{N_0})$, $r_r(\mathbf{y}) = (1 - \tilde{\rho}(\mathbf{y})) \log(1 + \frac{\gamma_{rd} P_R(\mathbf{y})}{N_0})$, $p(\mathbf{y}) = \rho(\mathbf{y}) P_{S1}(\mathbf{y}) + (\tilde{\rho}(\mathbf{y}) - \rho(\mathbf{y})) P_{S2}(\mathbf{y})$, $g(\mathbf{y}) = (\tilde{\rho}(\mathbf{y}) - \rho(\mathbf{y})) \eta \gamma_{sr} P_{S2}(\mathbf{y})$. In particular, as $N \rightarrow \infty$, we have $\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N r_{sr}(n) = \mathbb{E}_{\mathbf{y}}\{r_{sr}(\mathbf{y})\}$, $\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N r_{sd}(n) = \mathbb{E}_{\mathbf{y}}\{r_{sd}(\mathbf{y})\}$, $\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N r_{s1}(n) = \mathbb{E}_{\mathbf{y}}\{r_{s1}(\mathbf{y})\}$, $\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N r_r(n) = \mathbb{E}_{\mathbf{y}}\{r_r(\mathbf{y})\}$, $\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N p(n) = \mathbb{E}_{\mathbf{y}}\{p(\mathbf{y})\}$, $\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N g(n) = \mathbb{E}_{\mathbf{y}}\{g(\mathbf{y})\}$. Moreover, to make the problem tractable, we drop all causality constraints in (2) except for $m = N$, thus obtaining the following formulation for an upper bound on the ergodic capacity

$$\max_{\{P_{S1}(\mathbf{y}), P_{S2}(\mathbf{y}), \rho(\mathbf{y}), \tilde{\rho}(\mathbf{y}), P_R(\mathbf{y})\}} R \quad (5)$$

$$\text{s.t. } R \leq \mathbb{E}_{\mathbf{y}}\{r_{sr}(\mathbf{y}) + r_{sd}(\mathbf{y})\}, \quad (5a)$$

$$R \leq \mathbb{E}_{\mathbf{y}}\{r_{s1}(\mathbf{y}) + r_{sd}(\mathbf{y}) + r_r(\mathbf{y})\}, \quad (5b)$$

$$\mathbb{E}_{\mathbf{y}}\{p(\mathbf{y})\} \leq P, \mathbb{E}_{\mathbf{y}}\{(1 - \tilde{\rho}(\mathbf{y})) P_R(\mathbf{y})\} \leq \mathbb{E}_{\mathbf{y}}\{g(\mathbf{y})\}, \quad (5c)$$

$$\mathbb{E}_{\mathbf{y}}\{r_r(\mathbf{y})\} = \mathbb{E}_{\mathbf{y}}\{r_{sr}(\mathbf{y}) - r_{s1}(\mathbf{y})\}, \quad (5d)$$

$$0 \leq P_{S1}(\mathbf{y}), P_{S2}(\mathbf{y}) \leq P_p, \quad 0 \leq \rho(\mathbf{y}), \tilde{\rho}(\mathbf{y}) \leq 1.$$

The above problem can be converted into a convex problem and solved by minimizing the dual function, which will be discussed in Sec.III-B. The optimization result R in (5) gives an upper bound on R in (4). By applying the optimized variables in (5) to practical systems which consider the data and energy causality constraints in each relay transmission, we obtain an achievable rate. The practical transmission scheme is given at the end of Sec. III-B. Simulation results in Sec.V verify that the achievable rate is very close to the upper bound.

Partial CSI: The formulation above assumes that the source knows full CSI $\mathbf{y}$ so that it can schedule the transmission by the source or relay in each fading state. When the source only knows partial CSI $\mathbf{y}_s = [\gamma_{sr}, \gamma_{sd}]$ and has no feedback information from the relay, coordination between the source and relay cannot happen so that both the source and relay adapt the transmission parameters according to their own CSI. Consequently, the whole transmission process is divided into two phases. In the first phase, the source transmits information signals to the relay or destination with transmit power $P_{S1}(\mathbf{y}_s)$ or $P_{S2}(\mathbf{y}_s)$. In the second phase, the relay forwards information signals to the destination using the harvested energy with transmit power $P_R(\gamma_{rd})$.

As shown in Fig. 1(a), in the first phase, the source provides energy and data for the relay so that the relay has enough energy to forward data in the second phase. Therefore, in this case, data and energy causality constraints for $m < N$ in (2) are satisfied naturally. In the first phase, $\tilde{\rho}(\mathbf{y}) = 1$ and in the second phase, $\tilde{\rho}(\mathbf{y}) = 0$. The condition in (5d) degrades to $n_s(\bar{r}_{sr} - \bar{r}_{s1}) = n_r \bar{r}_r$ where $\bar{r}_{sr} = \mathbb{E}_{\mathbf{y}_s}\{r_{sr}(\mathbf{y}_s)\}$, $\bar{r}_{s1} = \mathbb{E}_{\mathbf{y}_s}\{r_{s1}(\mathbf{y}_s)\}$, $\bar{r}_r = \mathbb{E}_{\gamma_{rd}}\{r_r(\gamma_{rd})\}$. Therefore, $\frac{n_s}{n_r} = \frac{\bar{r}_r}{\bar{r}_{sr} - \bar{r}_{s1}}$ and the objective function $R = \frac{\bar{r}_{sd} + \bar{r}_{sr}}{1 + \frac{\bar{r}_{sd} + \bar{r}_{sr}}{\bar{r}_r}}$ where $\bar{r}_{sd} = \mathbb{E}_{\mathbf{y}_s}\{r_{sd}(\mathbf{y}_s)\}$. The problem in (5) can be simplified as

$$\max_{\{P_{S1}(\mathbf{y}_s), P_{S2}(\mathbf{y}_s), \rho(\mathbf{y}_s), P_R(\gamma_{rd})\}} R = \frac{\bar{r}_{sd} + \bar{r}_{sr}}{1 + \frac{\bar{r}_{sd} + \bar{r}_{sr}}{\bar{r}_r}} \quad (6)$$

$$\text{s.t. } \mathbb{E}_{\mathbf{y}_s}\{\rho(\mathbf{y}_s) P_{S1}(\mathbf{y}_s) + (1 - \rho(\mathbf{y}_s)) P_{S2}(\mathbf{y}_s)\} \leq P, \quad (6a)$$

$$\frac{\mathbb{E}_{\gamma_{rd}}\{P_R(\gamma_{rd})\}}{\mathbb{E}_{\mathbf{y}_s}\{(1 - \rho(\mathbf{y}_s)) \eta \gamma_{sr} P_{S2}(\mathbf{y}_s)\}} \leq \frac{\bar{r}_r}{\bar{r}_{sr} - \bar{r}_{s1}}, \quad (6b)$$

$$0 \leq P_{S1}(\mathbf{y}_s), P_{S2}(\mathbf{y}_s) \leq P_p, \quad 0 \leq \rho(\mathbf{y}_s) \leq 1. \quad (6c)$$

The problem in (6) is not convex due to the energy causality constraint (6b). The solution to (6) is described in the Sec.III-C.

B. Solution to the Full CSI Case

To solve (5), we first relax the constraint (5d) to $\mathbb{E}_{\mathbf{y}}\{r_r(\mathbf{y})\} \leq \mathbb{E}_{\mathbf{y}}\{r_{sr}(\mathbf{y}) - r_{s1}(\mathbf{y})\}$. It is easy to prove that the optimal solution without (5d) must satisfy $\mathbb{E}_{\mathbf{y}}\{r_r(\mathbf{y})\} \leq \mathbb{E}_{\mathbf{y}}\{r_{sr}(\mathbf{y}) - r_{s1}(\mathbf{y})\}$. Therefore, the relaxed constraint can be removed from (5). Note that $r_{sr}(\mathbf{y}), r_{sd}(\mathbf{y}), r_{s1}(\mathbf{y}), r_r(\mathbf{y})$ are not concave in $(P_{S1}(\mathbf{y}), P_{S2}(\mathbf{y}), \rho(\mathbf{y}), P_R(\mathbf{y}), \tilde{\rho}(\mathbf{y}))$. Therefore, we apply change of variables $\tilde{P}_{S1}(\mathbf{y}) = \rho(\mathbf{y}) P_{S1}(\mathbf{y})$, $\tilde{P}_{S2}(\mathbf{y}) = \tilde{\rho}(\mathbf{y}) P_{S2}(\mathbf{y})$, $\tilde{\rho}(\mathbf{y}) = \tilde{\rho}(\mathbf{y}) - \rho(\mathbf{y})$, $\tilde{P}_R(\mathbf{y}) = (1 - \tilde{\rho}(\mathbf{y})) P_R(\mathbf{y})$

to make $r_{sd}(\mathbf{y})$, $r_{sr}(\mathbf{y})$, $r_{s1}(\mathbf{y})$, $r_r(\mathbf{y})$ concave functions of variables. Therefore, the problem in (5) is re-expressed as

$$\max_{\{\tilde{P}_{S1}(\mathbf{y}), \tilde{P}_{S2}(\mathbf{y}), \rho(\mathbf{y}), \tilde{\rho}(\mathbf{y}), \tilde{P}_R(\mathbf{y})\}} R \quad (7)$$

$$s.t. R \leq \mathbb{E}_{\mathbf{y}}\{r_{sr}(\mathbf{y}) + r_{sd}(\mathbf{y})\}, \quad (7a)$$

$$R \leq \mathbb{E}_{\mathbf{y}}\{r_{s1}(\mathbf{y}) + r_{sd}(\mathbf{y}) + r_r(\mathbf{y})\}, \quad (7b)$$

$$\mathbb{E}_{\mathbf{y}}\{p(\mathbf{y})\} \leq P, \quad \mathbb{E}_{\mathbf{y}}\{\tilde{P}_R(\mathbf{y})\} \leq \mathbb{E}_{\mathbf{y}}\{g(\mathbf{y})\}, \quad (7c)$$

$$\tilde{\rho}(\mathbf{y}) - \rho(\mathbf{y}) = \bar{\rho}(\mathbf{y}), \quad 0 \leq \rho(\mathbf{y}), \tilde{\rho}(\mathbf{y}), \bar{\rho}(\mathbf{y}) \leq 1, \quad (7d)$$

$$0 \leq \tilde{P}_{S1}(\mathbf{y}) \leq \rho(\mathbf{y})P_p, \quad \tilde{P}_{S2}(\mathbf{y}) \leq \bar{\rho}(\mathbf{y})P_p, \quad \tilde{P}_R(\mathbf{y}) \geq 0. \quad (7e)$$

The Lagrangian associated with problem in (7) is

$$\begin{aligned} \mathcal{L} = & R - \lambda(\mathbb{E}_{\mathbf{y}}\{p(\mathbf{y})\} - P) - \mu(\mathbb{E}_{\mathbf{y}}\{\tilde{P}_R(\mathbf{y})\} - g(\mathbf{y})) \\ & - \nu(R - \mathbb{E}_{\mathbf{y}}\{r_{sr}(\mathbf{y}) + r_{sd}(\mathbf{y})\}) \\ & - \zeta(R - \mathbb{E}_{\mathbf{y}}\{r_{s1}(\mathbf{y}) + r_r(\mathbf{y}) + r_{sd}(\mathbf{y})\}) \\ = & \mathbb{E}_{\mathbf{y}}\{\mathcal{L}(\mathbf{y})\}, \end{aligned} \quad (8)$$

and the dual function is

$$\begin{aligned} l(\lambda, \mu, \nu, \zeta) = & \max_{\substack{\tilde{P}_{S1}(\mathbf{y}), \tilde{P}_{S2}(\mathbf{y}), \rho(\mathbf{y}), \\ \tilde{\rho}(\mathbf{y}), \bar{\rho}(\mathbf{y}), \tilde{P}_R(\mathbf{y})}} \mathbb{E}_{\mathbf{y}}\{\mathcal{L}(\mathbf{y})\} \\ s.t. & (7d), (7e). \end{aligned} \quad (9)$$

Since $\mathcal{L}$ is a linear function of R , we have $-1 + \nu + \zeta = 0$. Therefore, ζ can be eliminated by letting $\zeta = 1 - \nu$. The problem in (9) is a convex functional optimization problem. According to the Euler-Lagrange equation, the optimal $P_{S1}(\mathbf{y})$, $P_{S2}(\mathbf{y})$, $P_R(\mathbf{y})$ to maximize $\mathcal{L}$ are given by

$$\frac{\nu\gamma_{sr}/N_0}{1 + \gamma_{sr}P_{S1}^*(\mathbf{y})/N_0} + \frac{(1-\nu)\gamma_{sd}/N_0}{1 + \gamma_{sd}P_{S1}^*(\mathbf{y})/N_0} = \lambda \ln 2, \quad (10)$$

$$P_{S2}^*(\mathbf{y}) = \left[\frac{1}{\ln 2(\lambda - \mu\eta\gamma_{sr})} - \frac{1}{\gamma_{sd}/N_0} \right]_0^{P_p}, \quad (11)$$

$$P_R^*(\mathbf{y}) = \left[\frac{1-\nu}{(\ln 2)\mu} - \frac{1}{\gamma_{rd}/N_0} \right]^+, \quad (12)$$

where $[x]_a^b = a$, if $x < a$; $[x]_a^b = x$, if $a \leq x \leq b$; $[x]_a^b = b$, if $x > b$; $[x]^+ = \max(x, 0)$. The optimal $P_{S1}(\mathbf{y})$ is one of the solutions to (10). It can be proved that if $\lambda \ln 2 < \frac{\nu\gamma_{sr} + (1-\nu)\gamma_{sd}}{N_0}$, only one solution is positive and if $\lambda \ln 2 > \frac{\nu\gamma_{sr} + (1-\nu)\gamma_{sd}}{N_0}$, both solutions are negative. Denoting P_0 as the positive solution to (10), we have

$$P_{S1}^*(\mathbf{y}) = \begin{cases} 0, & \text{if } \lambda \ln 2 \geq c_1, \\ P_0, & \text{if } c_2 \leq \lambda \ln 2 < c_1, \\ P_p, & \text{if } \lambda \ln 2 \leq \min(c_1, c_2), \end{cases} \quad (13)$$

where $c_1 = \frac{\nu\gamma_{sr} + (1-\nu)\gamma_{sd}}{N_0}$, $c_2 = \frac{\nu\gamma_{sr}}{N_0 + \gamma_{sr}P_p} + \frac{(1-\nu)\gamma_{sd}}{N_0 + \gamma_{sd}P_p}$.

For any given $\mathbf{y}$, the partial derivatives $\frac{\partial \mathcal{L}(\mathbf{y})}{\partial \tilde{\rho}(\mathbf{y})}$, $\frac{\partial \mathcal{L}(\mathbf{y})}{\partial \rho(\mathbf{y})}$, $\frac{\partial \mathcal{L}(\mathbf{y})}{\partial \bar{\rho}(\mathbf{y})}$ are independent of $\tilde{\rho}(\mathbf{y})$, $\rho(\mathbf{y})$, $\bar{\rho}(\mathbf{y})$, respectively. Hence, the maximum of $\mathcal{L}(\mathbf{y})$ is achieved when $\{\tilde{\rho}(\mathbf{y}), \rho(\mathbf{y}), \bar{\rho}(\mathbf{y})\} \in \{0, 1\}$. Under the constraint (7d), all feasible solutions of $\{\tilde{\rho}(\mathbf{y}), \rho(\mathbf{y}), \bar{\rho}(\mathbf{y})\}$ include $(1, 0, 1)$, $(1, 1, 0)$, $(0, 0, 0)$. To find the optimal $\tilde{\rho}(\mathbf{y})$, $\rho(\mathbf{y})$, $\bar{\rho}(\mathbf{y})$, we need compare $\frac{\partial \mathcal{L}(\mathbf{y})}{\partial \tilde{\rho}(\mathbf{y})} = \log(1 + \frac{\gamma_{sd}P_{S2}^*(\mathbf{y})}{N_0}) - \frac{\gamma_{sd}P_{S2}^*(\mathbf{y})}{(\ln 2)(N_0 + \gamma_{sd}P_{S2}^*(\mathbf{y}))}$,

$\frac{\partial \mathcal{L}(\mathbf{y})}{\partial \rho(\mathbf{y})} = \nu \left(\log(1 + \frac{\gamma_{sr}P_{S1}^*(\mathbf{y})}{N_0}) - \frac{\gamma_{sr}P_{S1}^*(\mathbf{y})}{(\ln 2)(N_0 + \gamma_{sr}P_{S1}^*(\mathbf{y}))} \right) + (1 - \nu) \left(\log(1 + \frac{\gamma_{sd}P_{S1}^*(\mathbf{y})}{N_0}) - \frac{\gamma_{sd}P_{S1}^*(\mathbf{y})}{(\ln 2)(N_0 + \gamma_{sd}P_{S1}^*(\mathbf{y}))} \right)$, $-\frac{\partial \mathcal{L}(\mathbf{y})}{\partial \bar{\rho}(\mathbf{y})} = (1 - \nu) \left(\log(1 + \frac{\gamma_{rd}P_R^*(\mathbf{y})}{N_0}) - \frac{\gamma_{rd}P_R^*(\mathbf{y})}{(\ln 2)(N_0 + \gamma_{rd}P_R^*(\mathbf{y}))} \right)$ and choose the maximum one. Then, we have the following result

$$\begin{aligned} & \{\tilde{\rho}^*(\mathbf{y}), \rho^*(\mathbf{y})\} \\ = & \begin{cases} \{1, 0\}, & \text{if } \frac{\partial \mathcal{L}(\mathbf{y})}{\partial \tilde{\rho}(\mathbf{y})} > \max(\frac{\partial \mathcal{L}(\mathbf{y})}{\partial \rho(\mathbf{y})}, -\frac{\partial \mathcal{L}(\mathbf{y})}{\partial \bar{\rho}(\mathbf{y})}), \\ \{1, 1\}, & \text{if } \frac{\partial \mathcal{L}(\mathbf{y})}{\partial \rho(\mathbf{y})} > \max(\frac{\partial \mathcal{L}(\mathbf{y})}{\partial \tilde{\rho}(\mathbf{y})}, -\frac{\partial \mathcal{L}(\mathbf{y})}{\partial \bar{\rho}(\mathbf{y})}), \\ \{0, 0\}, & \text{if } -\frac{\partial \mathcal{L}(\mathbf{y})}{\partial \bar{\rho}(\mathbf{y})} > \max(\frac{\partial \mathcal{L}(\mathbf{y})}{\partial \tilde{\rho}(\mathbf{y})}, \frac{\partial \mathcal{L}(\mathbf{y})}{\partial \rho(\mathbf{y})}). \end{cases} \end{aligned} \quad (14)$$

1) *Summary of the Procedure for Solving (6):* First, we compute the channel independent parameters λ^* , ν^* , μ^* by minimizing the dual function $l(\lambda, \nu, \mu)$ subject to $\lambda \geq 0$, $\mu \geq 0$, $0 \leq \nu \leq 1$. This can be done by applying the subgradient-based method. The subgradient of $l(\lambda, \nu, \mu)$ at point (λ, μ, ν) is given by $P - \mathbb{E}_{\mathbf{y}}\{p\}$, $\mathbb{E}_{\mathbf{y}}\{g - \tilde{P}_R\}$, $\mathbb{E}_{\mathbf{y}}\{r_{sr} - r_{s1} - r_r\}$, where p , g , P_R , r_{sr} , r_{s1} , r_r are calculated by utilizing the optimal solution of (9) for given λ, μ, ν .

2) *Practical Transmission Scheme:* Assume at time n , the channel gain vector is $\mathbf{y}$. The source calculates $\{\tilde{\rho}^*(\mathbf{y}), \rho^*(\mathbf{y})\}$ utilizing known CSI. If $\tilde{\rho}^*(\mathbf{y}) = 1$, the source transmits to the relay or destination with power $P_{S1}^*(\mathbf{y})$ or $P_{S2}^*(\mathbf{y})$. Otherwise, the relay transmits with transmit power $P_R^*(\mathbf{y})$. The number of bits that the relay forwards depends on the CSI, data in the buffer and energy in the battery. The relay will stop transmitting when it uses up energy or has no data in buffer. Suppose the saved energy and data in the relay are $E(n)$ and $B(n)$, respectively. If the saved energy or data is not enough, i.e. $P_R^*(\mathbf{y})T < E(n)$ or $T \log(1 + \frac{\gamma_{rd}P_R^*(\mathbf{y})}{N_0}) \leq B(n)$ (T denotes the transmission period), to satisfy the causality constraints in each relay transmission, the transmission time should be adjusted to $t = \min(\frac{E(n)}{P_R^*(\mathbf{y})}, \frac{B(n)}{\log(1 + \frac{\gamma_{rd}P_R^*(\mathbf{y})}{N_0})})$.

C. Solution to the Partial CSI Case

To solve the problem in (6), we first solve for $P_R(\gamma_{rd})$ by fixing $P_{S1}(\mathbf{y}_s)$, $P_{S2}(\mathbf{y}_s)$, $\rho(\mathbf{y}_s)$. Then, given $P_R(\gamma_{rd})$ in terms of $P_{S1}(\mathbf{y}_s)$, $P_{S2}(\mathbf{y}_s)$, $\rho(\mathbf{y}_s)$, we find the optimal $P_{S1}(\mathbf{y}_s)$, $P_{S2}(\mathbf{y}_s)$, $\rho(\mathbf{y}_s)$ by fraction programming and convex optimization.

According to the definition of the convex functional [23], it is easy to prove that $\bar{r}_r$ is a concave functional of $P_R(\gamma_{rd})$. Therefore, for given $P_{S1}(\mathbf{y}_s)$, $P_{S2}(\mathbf{y}_s)$, $\rho(\mathbf{y}_s)$, $\bar{r}_r$ is concave and $\mathbb{E}_{\gamma_{rd}}\{P_R(\gamma_{rd})\}(\bar{r}_{sr} - \bar{r}_{s1}) + \mathbb{E}_{\gamma_{sr}}\{\eta(1 - \rho(\mathbf{y}_s))\gamma_{sr}P_{S2}(\mathbf{y}_s)\}(-\bar{r}_r)$ is convex. To maximize R over $P_R(\gamma_{rd})$, we first solve the following sub-problem

$$\begin{aligned} & \max_{P_R(\gamma_{rd}) \geq 0} \bar{r}_r \\ s.t. & \frac{\bar{r}_{sr} - \bar{r}_{s1}}{\bar{r}_r} \leq \frac{\mathbb{E}_{\mathbf{y}_s}\{(1 - \rho(\mathbf{y}_s))\eta\gamma_{sr}P_{S2}(\mathbf{y}_s)\}}{\mathbb{E}_{\gamma_{rd}}\{P_R(\gamma_{rd})\}}. \end{aligned} \quad (15)$$

The Lagrangian associated with (15) is $\mathcal{L} = \bar{r}_r - \mu[\mathbb{E}_{\gamma_{rd}}\{P_R(\gamma_{rd})\}(\bar{r}_{sr} - \bar{r}_{s1}) - \mathbb{E}_{\mathbf{y}_s}\{(1 - \rho(\mathbf{y}_s))\eta\gamma_{sr}P_{S2}(\mathbf{y}_s)\}\bar{r}_r]$

where $\mu \geq 0$. According to the Euler-Lagrange equation, the optimal $P_R(\gamma_{rd})$ to maximize $\mathcal{L}$ is

$$P_R^*(\gamma_{rd}) = \left[\underbrace{\frac{\mu \mathbb{E}_{\gamma_s} \{ (1-\rho) \eta \gamma_{sr} P_{S2} \} + 1}{(\ln 2) \mu (\bar{r}_{sr} - \bar{r}_{s1})}}_{C_r} - \frac{1}{\gamma_{rd}/N_0} \right]^+. \quad (16)$$

Utilizing (16), we have

$$\mathbb{E}_{\gamma_{rd}} \{ P_R(\gamma_{rd}) \} = C_r \exp \left(-\frac{N_0}{L_{rd} C_r} \right) - \frac{N_0}{L_{rd}} \int_{\frac{N_0}{C_r}}^{\infty} \frac{\exp(-x/L_{rd})}{x} dx \quad (17)$$

and

$$\bar{r}_r = \frac{1}{\ln 2} \int_{\frac{N_0}{C_r}}^{\infty} \frac{\exp(-x/L_{rd})}{x} dx. \quad (18)$$

To achieve the maximum transmission rate, the relay should use up the energy. Therefore, substituting (17) and (18) into the constraint in (15), we obtain

$$\ln 2 \left[\frac{\frac{C_r L_{rd}}{N_0} \exp(-\frac{N_0}{C_r L_{rd}})}{\int_{\frac{N_0}{C_r L_{rd}}}^{\infty} \frac{\exp(-x)}{x} dx} - 1 \right] = \frac{C_s L_{rd}}{N_0} \quad (19)$$

where $C_s = \frac{\mathbb{E}_{\gamma_s} \{ (1-\rho) \eta \gamma_{sr} P_{S2} \}}{(\bar{r}_{sr} - \bar{r}_{s1})}$. Since C_s is a function of C_r , we denote $\frac{N_0}{C_r L_{rd}} = f(\frac{N_0}{C_s L_{rd}})$.

Then, the primal problem in (6) is equivalent to

$$\begin{aligned} \max_{P_{S1}(\gamma_s), P_{S2}(\gamma_s), \rho(\gamma_s)} & \frac{(\bar{r}_{sd} + \bar{r}_{sr})}{1 + (\bar{r}_{sr} - \bar{r}_{s1}) / \frac{1}{\ln 2} \int_{f(\frac{N_0}{C_s L_{rd}})}^{\infty} \frac{\exp(-x)}{x} dx} \\ \text{s.t. } & (6a), (6c). \end{aligned} \quad (20)$$

The problem in (20) is a non-convex problem since the objective function is the ratio of two nonlinear functions. To solve it, we first introduce a new variable t and the corresponding equality constraint $\int_{f(\frac{N_0}{C_s L_{rd}})}^{\infty} \frac{\exp(-x)}{x} dx = t$ into (20). It can be proved that $\int_{f(\frac{N_0}{C_s L_{rd}})}^{\infty} \frac{\exp(-x)}{x} dx$ is a monotonic increasing function of C_s , and its range is $(0, \infty)$ (see Appendix A). Therefore, for any t , the equation $\int_{f(\frac{N_0}{C_s L_{rd}})}^{\infty} \frac{\exp(-x)}{x} dx = t$ has only one solution $C_s = q(t)$. It can be proved that the Hessian of $r_{sr}(\gamma_s) - r_{s1}(\gamma_s)$ is negative semidefinite. Therefore, $\bar{r}_{sr} - \bar{r}_{s1}$ is a concave functional of $(P_{S1}(\gamma_s), \rho(\gamma_s))$. For given t , the objective function is not quasiconcave since it is the ratio of two concave functions. To transform the objective function into a quasiconcave function, we introduce two more new variables $\bar{r}_{sr}, \bar{r}_{s1}$ and extend the range of $\bar{r}_{sr}, \bar{r}_{s1}$ by letting $\bar{r}_{sr} \leq \mathbb{E}_{\gamma_s} \{ r_{sr}(\gamma_s) \}, \bar{r}_{s1} \leq \mathbb{E}_{\gamma_s} \{ r_{s1}(\gamma_s) \}$.

On the other hand, we denote $u^* = \max \frac{\bar{r}_{sd} + \bar{r}_{sr}}{1 + \frac{(\bar{r}_{sr} - \bar{r}_{s1}) \ln 2}{t}}$ subject to the constraints in (20). Utilizing [20, Th. 1] which is borrowed from nonlinear fractional programming [24], the objective function in fractional form can be expressed as an equivalent subtractive form, e.g. $\bar{r}_{sd} + \bar{r}_{sr} - u^*(1 + \frac{(\bar{r}_{sr} - \bar{r}_{s1}) \ln 2}{t})$, which is concave due to the quasiconcave property of the

original function. Moreover, the optimal solution achieves the maximum u^* if and only if

$$\bar{r}_{sd}^* + \bar{r}_{sr}^* - u^*(1 + \frac{(\bar{r}_{sr}^* - \bar{r}_{s1}^*) \ln 2}{t^*}) = 0 \quad (21)$$

where $\bar{r}_{sd}^*, \bar{r}_{sr}^*, \bar{r}_{s1}^*, t^*$ denote the optimal solutions in the feasible set of (20).

Therefore, for a given parameter u , we need solve the following problem

$$R(u) = \max_{\{t \geq 0\}} \max_{\substack{\{\tilde{P}_{S1}(\gamma_s), \tilde{P}_{S2}(\gamma_s), \\ \rho(\gamma_s), \bar{r}_{sr}, \bar{r}_{s1}\}}} \bar{r}_{sd} + \bar{r}_{sr} - u(1 + \frac{(\bar{r}_{sr} - \bar{r}_{s1}) \ln 2}{t}) \quad (22)$$

$$\text{s.t. } \mathbb{E}_{\gamma_s} \{ \tilde{P}_{S1}(\gamma_s) + \tilde{P}_{S2}(\gamma_s) \} \leq P, \quad (22a)$$

$$C_s = \frac{\mathbb{E}_{\gamma_s} \{ \eta \gamma_{sr} \tilde{P}_{S2}(\gamma_s) \}}{\bar{r}_{sr} - \bar{r}_{s1}} = q(t), \quad (22b)$$

$$\begin{aligned} 0 &\leq \tilde{P}_{S1}(\gamma_s) \leq \rho(\gamma_s) P_p, \\ 0 &\leq \tilde{P}_{S2}(\gamma_s) \leq (1 - \rho(\gamma_s)) P_p, \end{aligned} \quad (22c)$$

$$\begin{aligned} 0 &< \bar{r}_{sr} \leq \mathbb{E}_{\gamma_s} \{ r_{sr} \}, \\ 0 &< \bar{r}_{s1} \leq \mathbb{E}_{\gamma_s} \{ r_{s1} \}, \quad 0 \leq \rho(\gamma_s) \leq 1, \end{aligned} \quad (22d)$$

where $\tilde{P}_{S1}(\gamma_s) = \rho(\gamma_s) P_{S1}(\gamma_s)$ and $\tilde{P}_{S2}(\gamma_s) = (1 - \rho(\gamma_s)) P_{S2}(\gamma_s)$.

Lemma 1: The optimal values of $\bar{r}_{s1}, \bar{r}_{sr}$ satisfy $\bar{r}_{s1}^* = \mathbb{E}_{\gamma_s} \{ r_{s1} \}, \bar{r}_{sr}^* = \mathbb{E}_{\gamma_s} \{ r_{sr} \}$.

Proof: It can be proved by contradiction. If $\bar{r}_{s1}^* < \mathbb{E}_{\gamma_s} \{ r_{s1} \}, \bar{r}_{sr}^* < \mathbb{E}_{\gamma_s} \{ r_{sr} \}, \bar{r}_{sr}^* - \bar{r}_{s1}^* = \Delta$, we can always increase the objective function by increasing both $\bar{r}_{sr}^*$ and $\bar{r}_{s1}^*$ with the same increment (keeping $\bar{r}_{sr}^* - \bar{r}_{s1}^* = \Delta$) until $\bar{r}_{s1}^* = \mathbb{E}_{\gamma_s} \{ r_{s1} \}$ or $\bar{r}_{sr}^* = \mathbb{E}_{\gamma_s} \{ r_{sr} \}$. For given u, t , if $\bar{r}_{sr}^* = \mathbb{E}_{\gamma_s} \{ r_{sr} \}$ and $\bar{r}_{s1}^* < \mathbb{E}_{\gamma_s} \{ r_{s1} \}$, the objective function can be increased further by letting $\bar{r}_{s1}^* = \mathbb{E}_{\gamma_s} \{ r_{s1} \}$ and increasing t .

For given u, t , the inner maximization in (22) is a concave problem, which can be solved via Lagrange dual decomposition. For the inner maximization in (22), the associated Lagrangian $\mathcal{L}$ is

$$\begin{aligned} \mathcal{L} = & -u + \bar{r}_{sd} + (1 - \frac{u \ln 2}{t} + \mu q(t) - v) \bar{r}_{sr} \\ & + (\frac{u \ln 2}{t} - \beta - \mu q(t)) \bar{r}_{s1} - \lambda [\mathbb{E}_{\gamma_s} \{ \tilde{P}_{S1} + \tilde{P}_{S2} \} - \bar{P}] \\ & - \mu \mathbb{E}_{\gamma_s} \{ \eta \gamma_{sr} \tilde{P}_{S2} \} + v \mathbb{E}_{\gamma_s} \{ r_{sr} \} + \beta \mathbb{E}_{\gamma_s} \{ r_{s1} \} \end{aligned} \quad (23)$$

where $\lambda \geq 0, v \geq 0, \beta \geq 0$ are Lagrange multipliers.

The dual function is

$$l(\lambda, \mu, v, \beta) = \max_{(22c), (22d)} \mathcal{L} \quad (24)$$

For a fixed set of Lagrange multipliers, the optimal solution can be obtained by applying the KKT conditions. Since $\mathcal{L}$ is a functional of variables, maximizing $\mathcal{L}$ involves the calculus variation of double integral. Utilizing Ostrogradskii equations [28], the power allocations for the source and relay are given respectively by

$$P_{S1}^*(\gamma_s) = \begin{cases} 0, & \text{if } \lambda \ln 2 \geq d_1, \\ P_0, & \text{if } d_2 \leq \lambda \ln 2 < d_1, \\ P_p, & \text{if } \lambda \ln 2 \leq \min(d_1, d_2), \end{cases} \quad (25)$$

$$P_{S2}^*(\gamma_s) = \left[\frac{1}{(\lambda + \mu \eta \gamma_{sr}) \ln 2} - \frac{1}{\gamma_{sd}/N_0} \right]_0^{P_p} \quad (26)$$

where $d_1 = \frac{\nu \gamma_{sr} + \beta \gamma_{sd}}{N_0}$, $d_2 = \frac{\nu \gamma_{sr}}{N_0 + \gamma_{sr} P_p} + \frac{\beta \gamma_{sd}}{N_0 + \gamma_{sd} P_p}$, P_0 is the positive solution to $\frac{\nu \gamma_{sr}/N_0}{1 + \gamma_{sr} P_{S1}^*(\gamma_s)/N_0} + \frac{\beta \gamma_{sd}/N_0}{1 + \gamma_{sd} P_{S1}^*(\gamma_s)/N_0} = \lambda \ln 2$. It can be observed that the water-level in (26) is inversely proportional to γ_{sr} to guarantee that the relay can harvest enough energy.

For a particular fading state, the partial derivative of $\mathcal{L}$ with respect to $\rho(\gamma_s)$ is

$$\frac{\partial \mathcal{L}}{\partial \rho(\gamma_s)} = \underbrace{l(\gamma_{sd} P_{S2}(\gamma_s))}_{f_2(P_{S2})} - \underbrace{[\nu l(\gamma_{sr} P_{S1}(\gamma_s)) + \beta l(\gamma_{sd} P_{S1}(\gamma_s))]}_{f_1(P_{S1})} \quad (27)$$

where $l(x) = \frac{x/N_0}{(\ln 2)(1+x/N_0)} - \log(1 + x/N_0)$. Since $\frac{\partial \mathcal{L}}{\partial \rho}$ is independent of ρ , it can be derived that

$$\rho^*(\gamma_s) = \begin{cases} 1, & \text{if } f_1(P_{S1}^*) < f_2(P_{S2}^*), \\ 0, & \text{otherwise.} \end{cases} \quad (28)$$

Obviously, $\mathcal{L}$ is a linear function of $\bar{r}_{sr}$ and $\bar{r}_{s1}$. The supremum of a linear function is ∞ , except in the special case when it is identically zero. By explicitly including the dual feasibility conditions as constraints, the Lagrange dual problem associated with (24) is

$$\begin{aligned} \min_{\lambda \geq 0, \nu \geq 0, \mu, \beta \geq 0} \quad & l(\lambda, \nu, \mu, \beta) \\ \text{s.t.} \quad & \mu q(t) - \nu + 1 - u \ln 2/t = 0, \\ & \beta - u \ln 2/t + \mu q(t) = 0. \end{aligned} \quad (29)$$

The equality constraints can be eliminated by substituting $\nu = 1 - (\frac{u \ln 2}{t} - \mu q(t))$ and $\beta = 1 - \nu$ into the dual function. Then, the problem in (29) is converted into an unconstrained problem,

$$\min_{\lambda \geq 0, \mu \in [\frac{1}{q(t)}(\frac{u \ln 2}{t} - 1), \frac{u \ln 2}{q(t)t}]} l(\lambda, \mu) = g_0 + \lambda g_\lambda + \mu g_\mu \quad (30)$$

where

$$g_0 = -u + \frac{u \ln 2}{t} \mathbb{E}_{\gamma_s} \{r_{s1} - r_{sr}\} + \mathbb{E}_{\gamma_s} \{r_{sd} + r_{sr}\}, \quad (31)$$

$$g_\lambda = \bar{P} - \mathbb{E}_{\gamma_s} \{\tilde{P}_{S1}(\gamma_s) + \tilde{P}_{S2}(\gamma_s)\}, \quad (32)$$

$$g_\mu = -\mathbb{E}_{\gamma_s} \{\eta \gamma_{sr} \tilde{P}_{S2}(\gamma_s)\} + q(t) \mathbb{E}_{\gamma_s} \{r_{sr} - r_{s1}\}. \quad (33)$$

As the inner maximization problem of (22) is concave, it is guaranteed that the primal optimal solution can be obtained by solving the dual problem (30) using the subgradient-based method, provided that the chosen step size is sufficiently small. The Lagrange multiplier update equations are

$$\lambda(\ell + 1) = [\lambda(\ell) + s(\ell)g_\lambda(\ell)]^+, \quad (34)$$

$$\mu(\ell + 1) = [\mu(\ell) + s(\ell)g_\mu(\ell)] \frac{\frac{u \ln 2}{q(t)t}}{\frac{1}{q(t)}(\frac{u \ln 2}{t} - 1)}, \quad (35)$$

where ℓ denotes the iteration number and $s(\ell)$ is a positive step size. We adopt the stepsize formula in [27] $s(\ell) = \frac{\hat{l}(\lambda(\ell), \mu(\ell)) - l(\lambda(\ell), \mu(\ell))}{\ell(|g_\lambda(\ell)|^2 + |g_\mu(\ell)|^2)}$ where $\hat{l}(\lambda(\ell), \mu(\ell)) = \min_{1 \leq i \leq \ell} l(\lambda(i), \mu(i))$.

Algorithm 1 Procedure for calculating $\lambda^*, \mu^*, u^*, t^*$

- 1 Given P, P_p , tolerance $\epsilon > 0$. Choose proper u_l, u_h to make sure $R(u_l) > 0$ and $R(u_h) < 0$;
 - 2 **repeat**
 $u_n = (u_l + u_h)/2$;
repeat
 $t := t + \delta t$;
repeat (for given t)
Generate N channel realizations;
In each realization, compute $P_{S1}^*, P_{S2}^*, \rho^*$ using (25)-(28) for given λ, μ ;
Update λ, μ using (34),(35);
until λ, μ converge to the prescribed accuracy;
until $t = (\ln 2) \mathbb{E}_{\gamma_{rd}} \{\log(1 + \gamma_{rd} P_p/N_0)\}$;
if $\max_t R(u_n) > 0$, $u_l = u_n$; **else**, $u_h = u_n$;
until $\max_t R(u_n) < \epsilon$.
-

When the optimal dual solutions λ^*, μ^* are obtained by the subgradient method, the corresponding optimal solution to problem (24) converges to the primal optimal solution to problem (22). It is difficult to get the closed-form expression of $\mathbb{E}_{\gamma_s} \{\cdot\}$ in (31)-(33) and therefore they are obtained by numerical calculation.

Summary of the Procedure for Solving (22): For given u, t , we compute the optimal dual solutions λ^*, μ^* to (30) by using subgradient-based method. Specifically, in each iteration, the optimal solution $P_{S1}(\gamma_s), P_{S2}(\gamma_s), \rho(\gamma_s)$ for a given set of Lagrange multipliers are obtained by (25),(26),(28). Then these solutions are used for calculating the subgradient $g_\lambda(\ell)$ and $g_\mu(\ell)$ and updating the Lagrange multipliers via (34) and (35). $R(u)$ is a strictly monotonic decreasing function in u , thus the optimal u^* that satisfies (21) can be found by bisection method or Dinkel-bach method in [26]. The procedure for computing the optimal $\lambda^*, \mu^*, u^*, t^*$ is summarized in Algorithm 1. Once they are obtained, the closed-form optimal solutions to (6) are given by $P_{S1}^*(\gamma_s)$ in (25), $P_{S2}^*(\gamma_s)$ in (26), $\rho^*(\gamma_s)$ in (28) and $P_R^*(\gamma_{rd})$ in (16).

Assume that u^*, t^* are optimal solutions obtained by Algorithm 1. For given u^*, t^* , the primal optimal $\bar{r}_{sr}^*$ and its dual optimal ν^* should satisfy the complementary slackness. Specifically, if $\nu^* > 0$, $\bar{r}_{sr}^* = \mathbb{E}_{\gamma_s} \{r_{sr}\}$ and the optimal solution to (22) is also the optimal solution to (20). Otherwise, we need calculate $\bar{r}_{sr}^*$ utilizing the constraint (22b). If $\bar{r}_{sr}^* < \mathbb{E}_{\gamma_s} \{r_{sr}\}$, (22) yields an upper bound on the optimal value of the problem (20). In this case, when we apply the optimal solutions to the practical systems, the ratio of the transmission time of the source and relay should be adjusted to $\frac{n'_s}{n'_r} = \frac{\bar{r}_{sr}^*}{\mathbb{E}_{\gamma_s} \{r_{sr} - r_{s1}\}}$ and the achievable rate is $\frac{\mathbb{E}_{\gamma_s} \{r_{sr} + r_{sd}\}}{1 + \frac{\mathbb{E}_{\gamma_s} \{r_{sr} - r_{s1}\}}{\bar{r}_{sr}^*}}$. The simulation curves in Sec.V represent the achievable rate.

IV. INFORMATION AND ENERGY TRANSFER IN RELAY SYSTEMS WITHOUT DIRECT LINK

A. Problem Formulations

When a direct path between the source and destination is unavailable, i.e., $\alpha = 0$, the source will spend dedicated

time to transfer wireless energy to the relay. The optimal solution can not be obtained explicitly by letting γ_{sd} in $\{P_{S1}^*, P_{S2}^*, \tilde{\rho}^*, \rho^*\}$ equal zero. Therefore, in this section, we study the information/energy scheduling and power control in the two-hop relay channel when the source has full or partial CSI.

Full CSI: In this network, the channel gain vector is expressed as $\mathbf{y} = \{\gamma_{sr}, \gamma_{rd}\}$. Similar to the analysis in Sec. III-A, we consider the formulation in the asymptotic regime of $N \rightarrow \infty$, where the variables are functions of the instantaneous channels only. When the source transmits information to the relay, the transmit power is $P_D(\mathbf{y})$ and the rate $r_s(\mathbf{y}) = \rho(\mathbf{y}) \log(1 + \frac{\gamma_{sr} P_D(\mathbf{y})}{N_0})$; and when it transmits energy to the relay, the transmit power is $P_E(\mathbf{y})$. The transmit power of the relay is $P_R(\mathbf{y})$ and the rate $r_r(\mathbf{y}) = (1 - \tilde{\rho}(\mathbf{y})) \log(1 + \frac{\gamma_{rd} P_R(\mathbf{y})}{N_0})$. The transmit power at the source, and the received energy at the relay are given respectively by $p(\mathbf{y}) = \rho(\mathbf{y}) P_D(\mathbf{y}) + (\tilde{\rho}(\mathbf{y}) - \rho(\mathbf{y})) P_E(\mathbf{y})$ and $g(\mathbf{y}) = (\tilde{\rho}(\mathbf{y}) - \rho(\mathbf{y})) \eta \gamma_{sr} P_E(\mathbf{y})$. The average and peak power constraints imposed on the source are

$$\mathbb{E}_{\mathbf{y}}\{p(\mathbf{y})\} \leq P, \quad 0 \leq P_D(\mathbf{y}), \quad P_E(\mathbf{y}) \leq P_p. \quad (36)$$

Let R be the ergodic capacity. The max-flow-min-cut bound gives

$$R \leq \min \{\mathbb{E}_{\mathbf{y}}\{r_s(\mathbf{y})\}, \mathbb{E}_{\mathbf{y}}\{r_r(\mathbf{y})\}\}. \quad (37)$$

We drop all causality constraints on the relay except for $m = N$, and obtain the following formulation for an upper bound on the ergodic capacity

$$\max_{\tilde{P}_D(\mathbf{y}), \tilde{P}_E(\mathbf{y}), P_R(\mathbf{y}), \rho(\mathbf{y}), \tilde{\rho}(\mathbf{y}), \tilde{p}(\mathbf{y})} R, \quad (38)$$

$$s.t. \quad R \leq \mathbb{E}_{\mathbf{y}}\{r_s(\mathbf{y})\}, \quad R \leq \mathbb{E}_{\mathbf{y}}\{r_r(\mathbf{y})\}, \quad (38a)$$

$$\mathbb{E}_{\mathbf{y}}\{p(\mathbf{y})\} \leq P, \quad (38b)$$

$$\mathbb{E}_{\mathbf{y}}\{r_r(\mathbf{y})\} \leq \mathbb{E}_{\mathbf{y}}\{r_s(\mathbf{y})\}, \quad \mathbb{E}_{\mathbf{y}}\{\tilde{P}_R(\mathbf{y})\} \leq \mathbb{E}_{\mathbf{y}}\{g(\mathbf{y})\}, \quad (38c)$$

$$\tilde{\rho}(\mathbf{y}) - \rho(\mathbf{y}) = \tilde{\rho}(\mathbf{y}), \quad 0 \leq \rho(\mathbf{y}), \tilde{\rho}(\mathbf{y}), \tilde{p}(\mathbf{y}) \leq 1, \quad (38d)$$

$$0 \leq \tilde{P}_D(\mathbf{y}) \leq \rho(\mathbf{y}) P_p, \quad \tilde{P}_E(\mathbf{y}) \leq \tilde{\rho}(\mathbf{y}) P_p, \quad \tilde{P}_R(\mathbf{y}) \geq 0$$

where $\tilde{P}_D(\mathbf{y}) = \rho(\mathbf{y}) P_D(\mathbf{y})$, $\tilde{P}_E(\mathbf{y}) = \tilde{\rho}(\mathbf{y}) P_E(\mathbf{y})$, $P_R(\mathbf{y}) = (1 - \tilde{\rho}(\mathbf{y})) P_R(\mathbf{y})$. It is easy to prove that even we remove constraint condition $\mathbb{E}_{\mathbf{y}}\{r_r(\mathbf{y})\} \leq \mathbb{E}_{\mathbf{y}}\{r_s(\mathbf{y})\}$ in (38c), the optimal solution must satisfy $R = \mathbb{E}_{\mathbf{y}}\{r_s(\mathbf{y})\} = \mathbb{E}_{\mathbf{y}}\{r_r(\mathbf{y})\}$. Therefore, $\mathbb{E}_{\mathbf{y}}\{r_s(\mathbf{y})\} \leq \mathbb{E}_{\mathbf{y}}\{r_r(\mathbf{y})\}$ in (38c) is a redundant condition and can be removed. Then, the problem in (38) is converted into a convex problem and solved by utilizing KKT conditions, which will be discussed in Sec.IV-B.

Partial CSI: In this case, the transmission process is divided into two phases. In the first phase, $\tilde{\rho}(\mathbf{y}) = 1$, and the source transmits information or energy signals to the relay with transmit power $P_D(\gamma_{sr})$ or $P_E(\gamma_{sr})$. In the second phase, $\tilde{\rho}(\mathbf{y}) = 0$ and the relay forwards information signals to the destination using the harvested energy with transmit power $P_R(\gamma_{rd})$. In this case, R achieves the maximum value when $n_s \tilde{r}_s = n_r \tilde{r}_r$, where $\tilde{r}_s = \mathbb{E}_{\gamma_{sr}}\{r_s(\gamma_{sr})\}$, $\tilde{r}_r = \mathbb{E}_{\gamma_{rd}}\{r_r(\gamma_{rd})\}$. Therefore, the objective function in (38) becomes $R = \frac{\tilde{r}_s \tilde{r}_r}{\tilde{r}_s + \tilde{r}_r}$.

Let $J = \frac{1}{R}$. When the source only knows partial CSI, the problem in (38) can be simplified as

$$\min_{P_D(\gamma_{sr}), P_E(\gamma_{sr}), \rho(\gamma_{sr}), P_R(\gamma_{rd})} J = \frac{1}{\tilde{r}_s} + \frac{1}{\tilde{r}_r} \quad (39)$$

$$s.t. \quad \mathbb{E}_{\gamma_{sr}}\{\rho(\gamma_{sr}) P_D(\gamma_{sr}) + (1 - \rho(\gamma_{sr})) P_E(\gamma_{sr})\} \leq P, \quad (39a)$$

$$\mathbb{E}_{\gamma_{rd}}\{P_R(\gamma_{rd})\} \tilde{r}_s \leq \mathbb{E}_{\gamma_{sr}}\{\eta(1 - \rho(\gamma_{sr})) \gamma_{sr} P_E(\gamma_{sr})\}, \quad (39b)$$

$$0 \leq P_D(\gamma_{sr}), P_E(\gamma_{sr}) \leq P_p, \quad 0 \leq \rho(\gamma_{sr}) \leq 1. \quad (39c)$$

The problem in (39) is not convex because the second constraint is not convex. We will solve (39) in Sec.IV-C.

B. Solution to the Full CSI Case

In this subsection, we assume that γ_{rd} is fed back to the source via a low-rate feedback link by the relay so that the source knows the channel gain vector $\mathbf{y}$ and can coordinate the transmission between the source and relay in each fading state. We first prove that when the source transfers energy to the relay, the optimal transmit power $P_E^*(\mathbf{y}) = P_p$ (see Appendix B). Therefore, the Lagrangian associated with the problem in (38) is

$$\begin{aligned} \mathcal{L} = & R - \mu(R - \mathbb{E}_{\mathbf{y}}\{r_s\}) - \zeta(R - \mathbb{E}_{\mathbf{y}}\{r_r\}) \\ & - \nu(\mathbb{E}_{\mathbf{y}}\{\tilde{P}_R\} - \mathbb{E}_{\mathbf{y}}\{g\}) - \lambda(\mathbb{E}_{\mathbf{y}}\{p\} - P) \\ = & \mathbb{E}_{\mathbf{y}}\{\mathcal{L}(\mathbf{y})\} \end{aligned} \quad (40)$$

where $\mu \geq 0, \zeta \geq 0, \nu \geq 0, \lambda \geq 0$ are Lagrange multipliers. For given Lagrange multipliers, $\mathcal{L}$ is a linear function of R . Therefore, to guarantee $\mathcal{L}$ bounded above, the ‘hidden’ constraint is $1 - \zeta - \mu = 0$. Utilizing the Euler-Lagrange equation, the optimal $P_D(\mathbf{y}), P_R(\mathbf{y})$ to maximize $\mathcal{L}$ are given by

$$P_D^*(\mathbf{y}) = \left[C_d - \frac{1}{\gamma_{sr}/N_0} \right]_0^{P_p}, \quad (41)$$

$$P_R^*(\mathbf{y}) = \left[C_r - \frac{N_0}{\gamma_{rd}} \right]^+ \quad (42)$$

where $C_d = \frac{\mu}{\lambda \ln 2}$, $C_r = \frac{1 - \mu}{\nu \ln 2}$.

For any given $\mathbf{y}$, the partial derivatives $\frac{\partial \mathcal{L}(\mathbf{y})}{\partial \rho(\mathbf{y})}, \frac{\partial \mathcal{L}(\mathbf{y})}{\partial \tilde{\rho}(\mathbf{y})}, \frac{\partial \mathcal{L}(\mathbf{y})}{\partial \tilde{p}(\mathbf{y})}$ are independent of $\tilde{\rho}(\mathbf{y}), \rho(\mathbf{y}), \tilde{p}(\mathbf{y})$, respectively. Hence, the maximum of $\mathcal{L}(\mathbf{y})$ is achieved when $\{\tilde{\rho}(\mathbf{y}), \rho(\mathbf{y}), \tilde{p}(\mathbf{y})\} \in \{0, 1\}$. The optimal $\{\tilde{\rho}^*(\mathbf{y}), \rho^*(\mathbf{y})\}$ which maximize $\mathcal{L}(\mathbf{y})$ under constraint (38d) are given in (14). In the case of no direct link, the result in (14) can be further simplified. Differentiating $\mathcal{L}(\mathbf{y})$ with respect to $\rho(\mathbf{y}), \tilde{\rho}(\mathbf{y})$ respectively, we obtain

$$\frac{\partial \mathcal{L}(\mathbf{y})}{\partial \rho(\mathbf{y})} = \mu \left(\log(1 + \frac{\gamma_{sr} P_D(\mathbf{y})}{N_0}) - \frac{1}{\ln 2} \frac{\gamma_{sr} P_D(\mathbf{y})/N_0}{1 + \gamma_{sr} P_D(\mathbf{y})/N_0} \right), \quad (43)$$

$$\frac{\partial \mathcal{L}(\mathbf{y})}{\partial \tilde{\rho}(\mathbf{y})} = \nu \eta \gamma_{sr} P_p - \lambda P_p. \quad (44)$$

Based on the result in (14), under the condition of $\tilde{\rho}(\mathbf{y}) = 1$, when $\mu \log(1 + \frac{\gamma_{sr} P_D^*}{N_0}) - \frac{\mu}{\ln 2} \frac{\gamma_{sr} P_D^*/N_0}{1 + \gamma_{sr} P_D^*/N_0} > \lambda(\frac{\nu \eta \gamma_{sr}}{\lambda} - 1) P_p$, $\rho^*(\mathbf{y}) = 1, \tilde{\rho}^*(\mathbf{y}) = 0$; otherwise, $\rho^*(\mathbf{y}) = 0, \tilde{\rho}^*(\mathbf{y}) = 1$. For clarify, we denote $C_e = \frac{\lambda}{\nu \eta}$. C_d and C_e represent the water levels for information transmit power and energy

Algorithm 2 Procedure for calculating the optimal C_d, x_0, C_r

```

1  Given  $P, P_p$ . Initial  $C_d^l, C_d^u, x_0^l, x_0^u, C_r^l, C_r^u$ ,
    $C_d = \frac{C_d^l + C_d^u}{2}, x_0 = \frac{x_0^l + x_0^u}{2}, C_r = \frac{C_r^l + C_r^u}{2}$ ;
2  while ( $|\mathbb{E}\{p\} - P| > \epsilon$ )
   while ( $|\mathbb{E}\{r_s\} - \mathbb{E}\{r_r\}| > \epsilon$ )
     while ( $|\mathbb{E}\{P_R\} - \mathbb{E}\{g\}| > \epsilon$ )
       Generate  $N$  channel realizations:
       For each given  $\gamma$ , calculate  $P_D^*, P_R^*$  with
       (41),(42); calculate  $\rho^*$  by comparing  $\gamma_{sr}/N_0$ 
       and  $x_0$ ; choose transmit node by calculating
        $\tilde{\rho}^*$  with (14).
       Calculate  $\mathbb{E}\{r_s\}, \mathbb{E}\{r_r\}, \mathbb{E}\{p\}, \mathbb{E}\{P_R\}, \mathbb{E}\{g\}$ ;
       If  $\mathbb{E}\{P_R\} < \mathbb{E}\{g\}$ ,  $C_r^l = C_r$ ; else  $C_r^u = C_r$ ;
        $C_r = \frac{C_r^l + C_r^u}{2}$  End;
       If  $\mathbb{E}\{r_s\} < \mathbb{E}\{r_r\}$ ,  $x_0^l = x_0$ ; else  $x_0^u = x_0$ ;
        $x_0 = \frac{x_0^l + x_0^u}{2}$  End;
       If  $\mathbb{E}\{p\} < P$ ,  $C_d^l = C_d$ ; else  $C_d^u = C_d$ ;  $C_d = \frac{C_d^l + C_d^u}{2}$ 
End

```

transmit power, respectively. The following Lemma gives a clear relationship between $\rho^*(\gamma)$ and channel gain γ_{sr} .

Lemma 2: If $C_e < \frac{N_0}{C_d}$, $\rho^*(\gamma) \equiv 0$ for any γ_{sr} . Otherwise, there exists an SNR threshold x_0 so that when $\frac{\gamma_{sr}}{N_0} > x_0$, $\rho^*(\gamma) = 0$; else, $\rho^*(\gamma) = 1$. x_0 is determined by the following rule:

When $C_d > P_p$, if $\frac{C_d}{P_p} \ln(\frac{C_d}{C_d - P_p}) < \frac{N_0}{C_e(C_d - P_p)}$, $x_0 \in [\frac{1}{C_d}, \frac{1}{C_d - P_p}]$ is the solution to $\frac{C_d}{P_p}(-\ln(C_d x) + 1 - \frac{1}{C_d x}) = 1 - \frac{N_0 x}{C_e}$; else, $x_0 > \frac{1}{C_d - P_p}$ is the solution to $\frac{C_d}{P_p}(-\ln(1 + x P_p) + \frac{x P_p}{1 + x P_p}) = 1 - \frac{N_0 x}{C_e}$.

When $C_d < P_p$, x_0 is the solution to $\frac{C_d}{P_p}(-\ln(C_d x) + 1 - \frac{1}{C_d x}) = 1 - \frac{N_0 x}{C_e}$.

Proof: See Appendix C.

Because the primal problem is convex, the optimal dual variables μ, ν, λ can be obtained by subgradient-based method, as stated in Section III-B. Since the subgradient-based method is time-consuming, we seek another way to locate the optimal μ, ν, λ which satisfy equality constraints in (38b) and (38c) quickly. Since μ, ν, λ can be obtained by one-to-one mapping $\lambda = \frac{\eta C_e}{(\eta C_d C_e + C_r) \ln 2}, \nu = \frac{1}{(\eta C_d C_e + C_r) \ln 2}, \mu = \frac{\eta C_d C_e}{\eta C_d C_e + C_r}$, we first get the optimal C_d, C_r, x_0 which satisfy equality constraints. C_e can be calculated utilizing the relationship among C_d, C_e, x_0 in Lemma 2. It is observed that the average transmit power of the source is an increasing function of C_d , the harvested energy of the relay is a decreasing function of x_0 , and the consumed energy of the relay is an increasing function of C_r . Therefore, we use the bisection method to find out the optimal C_d, C_r, x_0 .

Summary of the Procedure for Solving (38): We first compute the constants C_d, x_0, C_r using Algorithm 2. It is difficult to get the closed-form expression of $\mathbb{E}_\gamma\{\cdot\}$ in Algorithm 2 so that they are obtained by numerical calculation. Then the transmission node and transmission mode of the source are selected based on (14). If $\tilde{\rho}^*(\gamma) = 1$, $\rho^*(\gamma)$ is determined by

comparing $\frac{\gamma_{sr}}{N_0}$ and x_0 , and the closed-form optimal solutions to (38) are given by $P_E^*(\gamma) = P_p, P_D^*(\gamma)$ in (41), $P_R^*(\gamma) = 0$. If $\tilde{\rho}^*(\gamma) = 0$, the optimal solutions are given by $P_E^*(\gamma) = P_D^*(\gamma) = 0$, and $P_R^*(\gamma)$ in (42).

Recall that the formulation in (38) corresponds to an upper bound on the ergodic capacity since the data and energy causalities are ignored. As stated in the end of Sec. III-B, we can obtain achievable throughput by applying the optimal solution to a practical transmission scheme. The simulation results in Fig. 4 show that the gap between the upper bound and achievable rate is very small.

C. Solution to the Partial CSI Case

We first prove that in the optimal solution to (39), $P_E^*(\gamma_{sr}) = P_p$ (see Appendix B). Then, we solve for $P_R(\gamma_{rd})$ by fixing $P_D(\gamma_{sr}), \rho(\gamma_{sr})$. Finally, given $P_R(\gamma_{rd})$ in terms of $P_D(\gamma_{sr}), \rho(\gamma_{sr})$, we find the optimal $P_D(\gamma_{sr}), \rho(\gamma_{sr})$ by convex optimization and linear search.

To minimize J over $P_R(\gamma_{rd})$, we first solve the following sub-problem:

$$\begin{aligned} & \max_{P_R(\gamma_{rd}) \geq 0} \bar{r}_r, \\ & \text{s.t. } \mathbb{E}_{\gamma_{rd}}\{P_R(\gamma_{rd})\} \bar{r}_s \leq \mathbb{E}_{\gamma_{sr}}\{\eta(1 - \rho)\gamma_{sr} P_p\} \bar{r}_r. \end{aligned} \quad (45)$$

Since (45) is a concave optimization problem satisfying Slater's condition, we can apply KKT conditions to it. The Lagrangian associated with (45) is $\mathcal{L}(P_R, \mu) = \bar{r}_r - \mu[\mathbb{E}_{\gamma_{rd}}\{P_R(\gamma_{rd})\} \bar{r}_s - \mathbb{E}_{\gamma_{sr}}\{\eta(1 - \rho(\gamma_{sr}))\gamma_{sr} P_p\} \bar{r}_r]$ where $\mu > 0$ is the Lagrange multiplier. Since $\mathcal{L}(P_R, \mu)$ is a functional of $P_R(\gamma_{rd})$, by the Euler-Lagrange equation, we get

$$P_R^*(\gamma_{rd}) = \left[\frac{\frac{\mu \mathbb{E}_{\gamma_{sr}}\{\eta(1 - \rho(\gamma_{sr}))\gamma_{sr} P_p\} + 1}{\mu \bar{r}_s \ln 2} - \frac{1}{\gamma_{rd}/N_0}}{C_r} \right]^+. \quad (46)$$

To achieve the maximum transmission rate, the relay should use up the energy. Therefore, substituting (46) into the constraint in (45), we obtain the relationship between C_r and C_s , i.e., (19) where $C_s = \frac{\mathbb{E}_{\gamma_{sr}}\{\eta(1 - \rho(\gamma_{sr}))\gamma_{sr} P_p\}}{\bar{r}_s}$.

Substituting $\bar{r}_r$ into (39), the primal problem in (39) is then equivalent to

$$\begin{aligned} & \min_{P_D(\gamma_{sr}), \rho(\gamma_{sr})} \frac{1}{\bar{r}_s} + \frac{\ln 2}{\int_{f(\frac{N_0}{C_s L_{rd}})}^{\infty} \frac{\exp(-x)}{x} dx} \\ & \text{s.t. } (39a), (39c). \end{aligned} \quad (47)$$

The problem in (47) is not a convex problem since the second term of the objective function is not a convex function of $P_D(\gamma_{sr})$ and $\rho(\gamma_{sr})$. For any t , the equation $\int_{f(\frac{N_0}{C_s L_{rd}})}^{\infty} \frac{\exp(-x)}{x} dx = t$ has only one solution $C_s = q(t)$. To solve (47), we introduce two new variables $\bar{r}_s, t$, as well as an inequality constraint (49b) and an equality constraint (49c) into (47) and reformulate it as

$$\min_{t > 0} \frac{\ln 2}{t} + \frac{1}{J_1(t)} \quad (48)$$

where $J_1(t)$ is the maximum value of the objective function of the following problem:

$$J_1(t) = \max_{\tilde{P}_D(\gamma_{sr}), \rho(\gamma_{sr}), \tilde{r}_s} \tilde{r}_s \quad (49)$$

$$s.t. \mathbb{E}_{\gamma_{sr}} \{\tilde{P}_D(\gamma_{sr}) + (1 - \rho(\gamma_{sr}))P_p\} \leq P, \quad (49a)$$

$$\tilde{r}_s \leq \mathbb{E}_{\gamma_{sr}} \left\{ \rho(\gamma_{sr}) \log \left(1 + \frac{\gamma_{sr} \tilde{P}_D(\gamma_{sr})}{\rho(\gamma_{sr}) N_0} \right) \right\}, \quad (49b)$$

$$C_s = \frac{\mathbb{E}_{\gamma_{sr}} \{\eta(1 - \rho(\gamma_{sr}))\gamma_{sr} P_p\}}{\tilde{r}_s} = q(t), \quad (49c)$$

$$0 \leq \tilde{P}_D(\gamma_{sr}) \leq \rho(\gamma_{sr}) P_p, \quad (49d)$$

$$\tilde{r}_s > 0, \quad 0 \leq \rho(\gamma_{sr}) \leq 1$$

where $\tilde{P}_D(\gamma_{sr}) = \rho(\gamma_{sr}) P_D(\gamma_{sr})$. The problem in (48)-(49) and that in (47) are equivalent since the optimal objective value must be achieved when $\tilde{r}_s = \mathbb{E}_{\gamma_{sr}} \{\rho \log(1 + \gamma_{sr} P_D(\gamma_{sr})/N_0)\}$. For any given t , (49) is a convex optimization problem. The Lagrangian $\mathcal{L}$ associated with (49) is

$$\begin{aligned} \mathcal{L} = & \tilde{r}_s - \lambda(\mathbb{E}_{\gamma_{sr}} \{\tilde{P}_D + (1 - \rho)P_p\} - P) \\ & - \mu \left(\tilde{r}_s - \mathbb{E}_{\gamma_{sr}} \left\{ \rho \log \left(1 + \frac{\gamma_{sr} \tilde{P}_D}{\rho N_0} \right) \right\} \right) \\ & - \nu(\mathbb{E}_{\gamma_{sr}} \{\eta(1 - \rho)\gamma_{sr} P_p\} - q(t)\tilde{r}_s) \end{aligned} \quad (50)$$

where $\lambda \geq 0, \mu \geq 0$.

For given λ, μ, ν , we maximize $\mathcal{L}$ by optimizing $\tilde{P}_D(\gamma_{sr}), \rho(\gamma_{sr}), \tilde{r}_s$. $\mathcal{L}$ is a convex functional of $\tilde{P}_D(\gamma_{sr}), \rho(\gamma_{sr})$ and $\tilde{r}_s$. Therefore, if $\rho^*(\gamma_{sr}) \neq 0$, by solving the Euler-Lagrange equation, we can get the closed-form expression for the optimal $P_D(\gamma_{sr})$,

$$P_D^*(\gamma_{sr}) = \left[\underbrace{\frac{\mu}{\lambda \ln 2}}_{C_d} - \frac{1}{\gamma_{sr}/N_0} \right]_0^{P_p}. \quad (51)$$

On the other hand, if $\rho^*(\gamma_{sr}) = 0, P_D^*(\gamma_{sr}) = 0$. $\mathcal{L}$ is a linear functional of $\tilde{r}_s$. Therefore, $1 - \mu + \nu q(t) = 0$. For a particular fading state, the partial derivative of $\mathcal{L}$ with respect to $\rho(\gamma_{sr})$ is

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \rho(\gamma_{sr})} = & -\lambda P_p - \mu \log \left(1 + \frac{\gamma_{sr} P_D}{N_0} \right) + \frac{\mu}{\ln 2} \frac{\gamma_{sr} P_D/N_0}{1 + \gamma_{sr} P_D/N_0} \\ & - \nu \eta P_p \gamma_{sr}. \end{aligned} \quad (52)$$

Applying the analysis in Sec.IV-B, we can find a threshold x_0 . When $\frac{\gamma_{sr}}{N_0} > x_0, \rho^*(\gamma_{sr}) = 0$; else, $\rho^*(\gamma_{sr}) = 1$.

Finally, we will obtain the optimal C_d, x_0 which make the equalities in (49a) and (49c) hold. We propose an algorithm based on bisection method to solve the two nonlinear equations (see Appendix D). As shown in Fig. 2, we first need to determine the range of C_d, x_0 , and then search the optimal value in the corresponding range. Due to $t = \tilde{r}_r \ln 2$, we search the optimal t which maximizes (48) in the region of $(0, (\ln 2)\mathbb{E}_{\gamma_{rd}} \{\log(1 + \gamma_{rd} P_p/N_0)\})$. The overall algorithm for calculating the optimal C_d, x_0 is summarized in Algorithm 3.

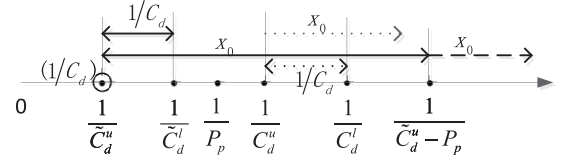


Fig. 2. Range of (C_d, x_0) in different cases: dot ($C_d < P_p$); dash ($C_d > P_p, x_0 \geq 1/(C_d - P_p)$); solid ($C_d > P_p, x_0 \in [1/C_d, 1/(C_d - P_p)]$). Boundary points of C_d are calculated in Appendix D.

Algorithm 3 Procedure for calculating the optimal C_d, x_0

1. Given P, P_p , search range $t \in [0, (\ln 2)\mathbb{E}_{\gamma_{rd}} \{\log(1 + \gamma_{rd} P_p/N_0)\}]$, and search step δt ;
repeat
2. For given t , calculate C_d, x_0 by using the flow diagram in Fig. 8; calculate $P_D^*(\gamma_{sr})$ in (51); calculate $\rho^*(\gamma_{sr})$ by comparing γ_{sr}/N_0 and x_0 ; calculate $\tilde{r}_s$;
3. update $t \leftarrow t + \delta t$;
- until** $t \geq (\ln 2)\mathbb{E}_{\gamma_{rd}} \{\log(1 + \gamma_{rd} P_p/N_0)\}$.
4. Select t^* that maximizes $\tilde{r}_s$ and output the corresponding C_d, x_0 .

Summary of the Procedure for Solving (39): We first compute the constants C_d, x_0 using Algorithm 3. We then compute the constant C_r in (46). Note that these constants are independent of the CSI. Then the closed-form optimal solution to (39) is given by $P_E^*(\gamma_{sr}) = P_p, P_D^*(\gamma_{sr})$ in (51), $\rho^*(\gamma_{sr}) = 1$ (if $\frac{\gamma_{sr}}{N_0} < x_0$) or 0 (if $\frac{\gamma_{sr}}{N_0} > x_0$), and $P_R^*(\gamma_{rd})$ in (46).

V. NUMERICAL RESULTS

In this section, we evaluate the throughput performance of the wireless-powered relay system when the source knows partial or full CSI. The bandwidth is set to 10MHz. The AWGN at the receiver is assumed to have a white power spectral density of -150dBm/Hz . The energy harvesting efficiency is assumed to be $\eta = 0.8$. The path loss in S-R, R-D, S-D links are modeled as $10^{-2}d_{sr}^{-2}, 10^{-2}d_{rd}^{-2}, 10^{-2}d_{sd}^{-2}$.

Fig. 3 presents the throughput gain of relay-assisted transmission over direct transmission without relay. In Fig. 3, $P_p = 1.2w, d_{sr} = 1m, d_{rd} = 4m, d_{sd} = 5m$. P is the average transmit power constraint in (6a) or (39a). Note that in the partial CSI case, P in (6a) (or (39a)) denotes the average transmit power of the source in the first phase, and the average transmit power of the source in the whole transmission process is $\frac{P}{1+n_r/n_s}$. Therefore, for a fair throughput comparison between full CSI case and partial CSI case, the average transmit power constraint of source in the full CSI case is made to equal $\frac{P}{1+n_r/n_s}$. It can be seen that the system with full CSI at the source outperforms that with partial CSI due to the adaptive scheduling between the source and relay. Because of the low power transfer efficiency, the relay-assisted transmission shows advantage over the direct transmission only when the direct link is weak. When α is small, more than half bits are forwarded by the relay. Therefore, the throughput gain (ratio of average rate of relay transmission to average rate

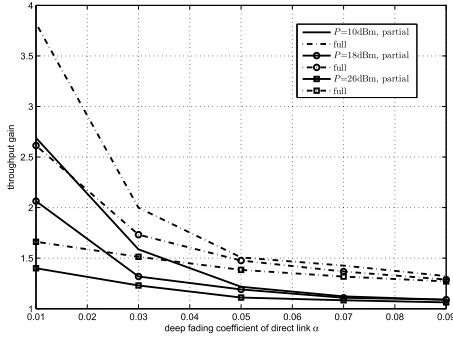


Fig. 3. Throughput gain vs. deep fading coefficient of direct link.

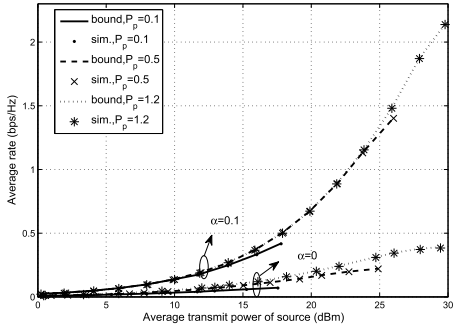
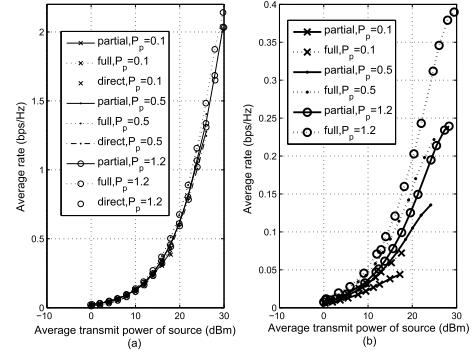
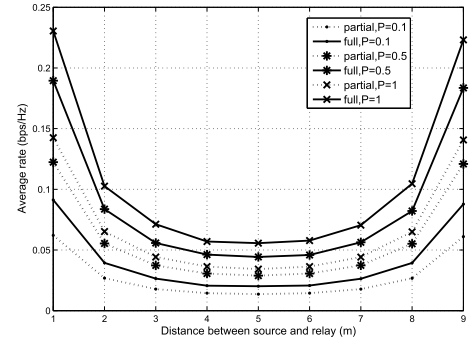


Fig. 4. Comparison of upper bound on rate and achievable rate (full CSI case).

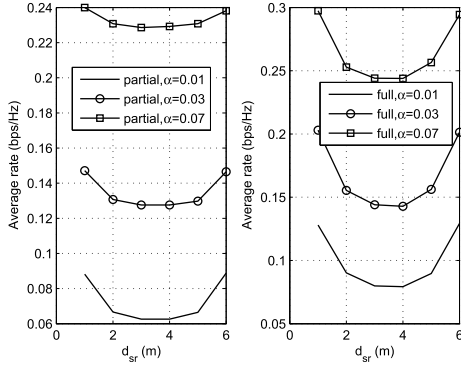
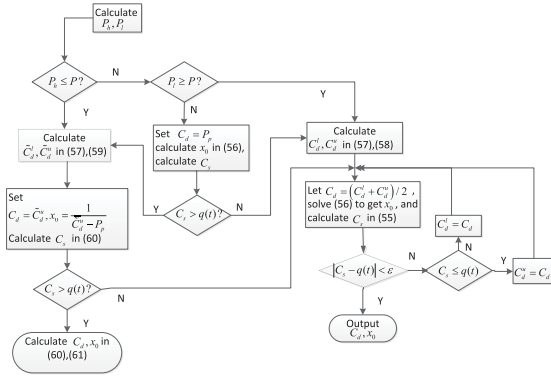
of direct transmission) is the highest when $\alpha = 0.01$. With the increase of α , the gain approaches 1 since more bits are transmitted to the destination via the direct link. On the other hand, the probability that the direct transmission is adopted increases as the average transmit power P increases. As a result, the throughput improvement from the wireless powered relay is more significant in the region of low transmit power. When $\alpha > 0.05$, the impact of average transmit power on the throughput gain is marginal.

When the source knows full CSI, we obtain an upper bound on the ergodic capacity by neglecting the causality constraints during the transmission. We apply the optimal solutions to the practical system to get an achievable rate. Fig. 4 compares the upper bound and achievable rate in systems with different configurations. It can be observed that in both the system with $\alpha = 0.1$ and that without direct link, the achievable rate is very close to the upper bound. In Fig. 5, we show the average throughput versus the average transmit power in both full CSI case and partial CSI case with different P_p . We set $d_{sr} = 2m$, $d_{rd} = 3m$, $d_{sd} = 5m$, $\alpha = 0.1$. The x-coordinate denotes the average transmit power of source in the whole transmission process, which is set to $0 \sim 18\text{dBm}$, $0 \sim 26\text{dBm}$, $0 \sim 30\text{dBm}$ in the case of $P_p = 0.1w$, $0.5w$, $1.2w$, respectively. When the direct link is unavailable, switching threshold x_0 decreases with the increase of the average or peak power of the source. As a result, the energy transmission process is shortened since the relay has more opportunities to harvest energy. In the low transmit power region ($< 15\text{dBm}$), the scheme with full CSI only obtains slight capacity improvement compared to the scheme with partial CSI. With the increase

Fig. 5. Average throughput vs. average transmit power of the source: (a) weak S-D link with $\alpha = 0.1$; (b) no direct link.Fig. 6. Average throughput vs. distance between source and relay ($\alpha = 0$).

of the transmit power, the capacity improvement due to the transmit node selection is more obvious. When α increases from 0 to 0.1, the throughput gap between partial CSI and full CSI decreases. As illustrated in Fig. 3, when $\alpha > 0.09$, the throughput gain achieved by the wireless powered relay is marginal. Therefore, the system with partial CSI and that with full CSI have similar throughput performance. Moreover, in the low transmit power region, both P_{S1} and P_{S2} hardly reach the peak value. Therefore, systems with different P_p have similar throughput performance. On the other hand, when the average transmit power of source increases, the probability that the source transmits to the destination via the direct link increases so that the relay can harvest more energy to forward more data.

The position of the relay has an important influence on the average rate. In Fig. 6, we set $P_p = 1.2w$, $d_{sd} = 10m$. P denotes the average transmit power constraint in (39a). For a fair comparison, in each point, we make the average transmit power of the source in the case of full CSI equal $\frac{P}{1+n_r/n_s}$. It is observed from Fig. 6 that the system achieves the minimum throughput when $d_{sr} \approx d_{rd}$. The throughput increases gradually when the relay approaches the source or the destination. This is because when the relay is close to the source or the destination, the source needs less time for transferring energy. In Fig. 7, we set $P_p = 1.2w$, $d_{sd} = 7m$, $P = 18\text{dBm}$. Similar to the phenomenon in Fig. 6, the system achieves the minimum throughput when $d_{sr} \approx d_{rd}$. In the system with partial CSI at source, the influence of the position of relay on the average rate weakens with the

Fig. 7. Average throughput vs. distance between the source and relay ($\alpha > 0$).Fig. 8. Flow diagram for solving C_d, x_0 .

increasing of α . In contrast, in the system with full CSI at source, the position of relay has similar effect on the average rate when $\alpha = 0.01, 0.03, 0.07$.

VI. CONCLUSIONS

In this paper, we consider a three-node relay system with a deep-fading direct link. The relay node harvests energy from RF signals transmitted by the source. Under a time-varying relay channel setup, whether the source knows full or partial CSI leads to different cooperative schemes. When the source knows full CSI, it can schedule the transmission between the source and relay and control the transmit power based on instantaneous CSI. An upper bound on ergodic throughput and corresponding transmission solutions are obtained by neglecting the causality constraints during the transmission. When the source only knows partial CSI, the transmission process is divided into two phases and the ratio of the transmission time of the two phases depends on statistical CSI. Both the source and relay adjust their transmit power according to known CSI. In a simplified network topology without direct link, the optimal transmission solutions are more concise. The parameters in optimal solutions are found by proposed algorithms based on bisection method. For all four problems considered in this paper, we have obtained the closed-form optimal solutions as functions of the CSI. Simulation results reveal the throughput gain brought by wireless powered relay in both full and partial cases, and demonstrate the influence of relay position on throughput performance.

APPENDICES

A. Property of Function $\tilde{f}(C_s) = \int_{f(\frac{N_0}{C_s L_{rd}})}^{\infty} \frac{\exp(-x)}{x} dx$

We first derive the following result: $f(\frac{N_0}{C_s L_{rd}})$ is a decreasing function of C_s .

Proof: Define $h(f) = \frac{\exp(-f)}{f}$ and $g(f) = \frac{h(f)}{\int_f^\infty h(y)dy} - 1$ ($g(f) = \frac{L_{rd} C_s}{N_0 \ln 2}$). The derivative of $g(f)$ is $g'(f) = \frac{h'(f) \int_f^\infty h(y)dy + h^2(f)}{(\int_f^\infty h(y)dy)^2} = \frac{-(1+\frac{1}{f})h(f) \int_f^\infty h(y)dy + h^2(f)}{(\int_f^\infty h(y)dy)^2} = \frac{h(f)\tilde{h}(f)}{(\int_f^\infty h(y)dy)^2}$. Due to $\tilde{h}'(f) = \frac{\int_f^\infty h(y)dy}{f^2} > 0$, $\tilde{h}(f)$ is an increasing function of f . Hence, $\tilde{h}(f) < \tilde{h}(\infty) = 0$, which results in $g'(f) < 0$. Therefore, $f(\frac{N_0}{C_s L_{rd}})$ is the decreasing function of C_s .

On the other hand, $\tilde{f}(C_s)$ is the decreasing function of f . Therefore, $\tilde{f}(C_s)$ is an increasing function of C_s .

B. Proof of $P_E^* = P_P$

Proof: It can be observed that for any a set of feasible solution $\{P_E, P_D, P_R, \rho, \tilde{\rho}\}$, we can find another feasible solution $\{\hat{P}_E, \hat{P}_D, \hat{P}_R, \hat{\rho}, \hat{\tilde{\rho}}\}$ by letting $\hat{P}_E = P_P, (\hat{\tilde{\rho}} - \hat{\rho})P_P = (\tilde{\rho} - \rho)P_E, \hat{\rho}\hat{P}_D = \rho P_D, \hat{P}_R = P_R, \hat{\tilde{\rho}} = \tilde{\rho}$. Comparing the two sets of solution, we have $\hat{\rho} \geq \rho$. By keeping ρP_D constant, r_s is an increasing function of ρ (it is easy to verify that $\frac{\partial r_s}{\partial \rho} > 0$). Therefore, $\hat{\rho} \geq \rho$ results in $\hat{r}_s \geq r_s$. Since the second set of solution can always achieve the higher transmission rate, the feasible set can be reduced by only including the feasible solutions with $P_E = P_P$.

C. Proof of Lemma 1

To find out the relationship between the optimal $\rho^*(\gamma)$ and channel gain γ_{sr} from (43) and (44), we consider the following three cases:

Case 1): $C_e > \frac{N_0}{C_d - P_P}$

Define $x = \frac{\gamma_{sr}}{N_0}$, $f_1(x) = C_d[-\ln(1 + x P_D^*) + \frac{x P_D^*}{1 + x P_D^*}]$, and $f_2(x) = (1 - \frac{N_0 x}{C_e})P_P$. When $\frac{1}{C_d} \leq x \leq \frac{1}{C_d - P_P}$, $P_D^* = C_d - \frac{1}{x}$, and

$$f_1(x|P_D^* < P_P) = C_d \left[-\ln(C_d x) + 1 - \frac{1}{C_d x} \right]. \quad (53)$$

It can be proved that $f_1(x|P_D^* < P_P)$ is an decreasing function of x in the range of $x \in [\frac{1}{C_d}, \frac{1}{C_d - P_P}]$, and when $x = \frac{1}{C_d}$, (53) achieves the maximum value 0. When $x \geq \frac{1}{C_d - P_P}$, $P_D^* = P_P$ and

$$f_1(x|P_D^* = P_P) = C_d \left[-\ln(1 + x P_P) + \frac{x P_P}{1 + x P_P} \right]. \quad (54)$$

Similarly, it can be proved that (54) is a decreasing convex function of x . When $x = \frac{1}{C_d - P_P}$, (53) is equal to (54). On the other hand, $f_2(x)$ is a decreasing function of x . Since $\frac{C_e}{N_0} > \frac{1}{C_d - P_P}$, the intersection x_0 is determined by the equation $f_1(x|P_D^* = P_P) = f_2(x)$. Because $\lim_{x \rightarrow \infty} f_1'(x|P_D^* = P_P) = 0$, it is can be observed from geometry that $f_1(x|P_D^* = P_P) = f_2(x)$ has only one solution $x_0 > \frac{C_e}{N_0}$.

In summary, in this case, when $x < x_0$, $f_1(x) < f_2(x)$ and $\rho^* = 1$; when $x > x_0$, $f_1(x) > f_2(x)$ and $\rho^* = 0$. The threshold x_0 is the solution of $f_1(x|P_D^* = P_p) = f_2(x)$.

Case 2): $\frac{N_0}{C_d} \leq C_e \leq \frac{N_0}{C_d - P_p}$

In this case, x_0 can be determined by $f_1(x|P_D^* < P_p) = f_2(x)$ or $f_1(x|P_D^* = P_p) = f_2(x)$. If $f_1(\frac{1}{C_d - P_p}|P_D^* < P_p) > f_2(\frac{1}{C_d - P_p})$, x_0 is determined by $f_1(x|P_D^* < P_p) = f_2(x)$. Otherwise, x_0 is determined by $f_1(x|P_D^* = P_p) = f_2(x)$. Meanwhile, we have the following result about the uniqueness of x_0 .

Lemma: If $f_1(\frac{1}{C_d - P_p}|P_D^* < P_p) > f_2(\frac{1}{C_d - P_p})$, $f_1(x|P_D^* < P_p) = f_2(x)$ has only one solution and $f_1(x|P_D^* = P_p) = f_2(x)$ has no solution.

Proof: Define $\tilde{f}_1(x) = f_1(x|P_D^* < P_p) - f_2(x)$. $\tilde{f}_1'(x) = -\frac{C_d}{x} + \frac{1}{x^2} + \frac{N_0 P_p}{C_e} > -\frac{1+P_p}{x} + \frac{1}{x^2} + \frac{N_0 P_p}{C_e} = -\frac{P_p}{x} + \frac{N_0 P_p}{C_e} > 0$. Hence, $\tilde{f}_1(x)$ is an increasing function of x when $x \in [\frac{C_e}{N_0}, \frac{1}{C_d - P_p}]$. Consequently, $\tilde{f}_1(x) = 0$ has only one solution. On the other hand, $\tilde{f}_2(x) = f_1(x|P_D^* = P_p) - f_2(x)$ is an increasing function of x . $\tilde{f}_2(\frac{1}{C_d - P_p}) > 0$ results in $\tilde{f}_2(x) > \tilde{f}_2(\frac{1}{C_d - P_p}) > 0$ in the range of $x > \frac{1}{C_d - P_p}$.

Case 3): $C_e < \frac{N_0}{C_d}$

In this case, $\tilde{f}_1(\frac{1}{C_d}) > 0$. According to the analysis in case 2, we can derive that when $x \in [\frac{1}{C_d}, \frac{1}{C_d - P_p}]$, $\tilde{f}_1(x) > \tilde{f}_1(\frac{1}{C_d}) > 0$, and when $x > \frac{1}{C_d - P_p}$, $\tilde{f}_2(x) > \tilde{f}_2(\frac{1}{C_d - P_p}) > 0$. Therefore, in Case 3, $\rho^* \equiv 0$.

D. Calculation of C_d, x_0

As shown in Fig. 2, we first need to judge whether C_d is greater or smaller than P_p according to the average power with $C_d = P_p$. When $C_d = P_p$, the minimum and maximum average powers are $P_l = \int_{\frac{N_0}{P_p}}^{\infty} (P_p - \frac{N_0}{x}) \frac{\exp(-x/L_{sr})}{L_{sr}} dx$ and $P_h = \int_{\frac{N_0}{P_p}}^{\infty} \frac{P_p \exp(-x/L_{sr})}{L_{sr}} dx$, respectively. Therefore, if $P_l > P$, $C_d \leq P_p$ and if $P_h < P$, $C_d \geq P_p$. If $P_l \leq P \leq P_h$, we calculate x_0 with (56), and then substitute x_0 into (55) to get C_s . If $C_s < q(t)$, $C_d < P$, else $C_d > P$.

Then, we discuss the following two cases:

1) $C_d \leq P_p$

In this case, the constraint $P_D(\gamma_{sr}) \leq P_p$ is inactive. Therefore, when $\frac{\gamma_{sr}}{N_0} < x_0$, $\rho^*(\gamma_{sr}) = 1$ and $P_D^*(\gamma_{sr}) = [C_d - \frac{N_0}{\gamma_{sr}}]^+$; else, $\rho^*(\gamma_{sr}) = 0$. The constraints in (49a) and (49c) can be written as

$$\frac{\eta P_p (N_0 x_0 + L_{sr}) \exp(-N_0 x_0 / L_{sr})}{\int_{\frac{N_0}{C_d}}^{\frac{N_0}{x_0}} \log_2(C_d x / N_0) \frac{\exp(-x/L_{sr})}{L_{sr}} dx} = q(t), \quad (55)$$

$$\int_{\frac{N_0}{C_d}}^{\frac{N_0}{x_0}} (C_d - \frac{N_0}{x}) \frac{\exp(-x/L_{sr})}{L_{sr}} dx + \int_{N_0 x_0}^{\infty} \frac{P_p \exp(-x/L_{sr})}{L_{sr}} dx = P. \quad (56)$$

Since $C_d - \frac{N_0}{\gamma_{sr}} < P_p$, it can be observed from (56) that for given C_d , the average transmit power is a decreasing function of x_0 . Therefore, when $x_0 = \frac{1}{C_d}$, P achieves the maximum

value $P_p \exp(-\frac{N_0}{C_d L_{sr}})$ and the lower boundary point of C_d is decided by

$$P_p \exp(-\frac{N_0}{C_d L_{sr}}) = P. \quad (57)$$

When $x_0 \rightarrow \infty$, the average power achieves the minimum value $\int_{\frac{N_0}{C_d}}^{\infty} (C_d - \frac{N_0}{x}) \frac{\exp(-x/L_{sr})}{L_{sr}} dx$ and the upper boundary point of C_d is decided by

$$\int_{\frac{N_0}{C_d}}^{\infty} (C_d - \frac{N_0}{x}) \frac{\exp(-x/L_{sr})}{L_{sr}} dx = P. \quad (58)$$

On the other hand, for any (C_d, x_0) that satisfies (56), C_s is a decreasing function of C_d and when $C_d \in [C_d^l, C_d^u]$, $C_s \in (0, \infty)$. Therefore, we can use the bisection method to find C_d which makes $C_s = q(t)$.

2) $C_d \geq P_p$

Based on the interval that x_0 lies, we consider the following two cases further:

2.1) $x_0 \in [\frac{1}{C_d}, \frac{1}{C_d - P_p}]$.

In this case, the constraints can be expressed as (55)-(56). When $x_0 = \frac{1}{C_d}$, the average transmit power achieves the maximum value and the lower boundary point of C_d is decided by (57). When $x_0 = \frac{1}{C_d - P_p}$, the average transmit power achieves the minimum value and the upper boundary point of C_d is decided by

$$\int_{\frac{N_0}{C_d}}^{\frac{N_0}{C_d - P_p}} (\tilde{C}_d - \frac{N_0}{x}) \frac{\exp(-x/L_{sr})}{L_{sr}} dx + \int_{\frac{N_0}{C_d - P_p}}^{\infty} \frac{P_p \exp(-x/L_{sr})}{L_{sr}} dx = P. \quad (59)$$

When $C_d \in [\tilde{C}_d^l, \tilde{C}_d^u]$, we use the bisection method to find C_d which makes $C_s = q(t)$.

2.2) $x_0 \geq \frac{1}{C_d - P_p}$.

In this case, the constraints in (49a) and (49c) are given by

$$\eta P_p (N_0 x_0 + L_{sr}) \exp(-N_0 x_0 / L_{sr}) / \left[\int_{\frac{N_0}{C_d}}^{\frac{N_0}{C_d - P_p}} \log_2(C_d x / N_0) \frac{\exp(-x/L_{sr})}{L_{sr}} dx + \int_{\frac{N_0}{C_d - P_p}}^{N_0 x_0} \log_2(1 + P_p x / N_0) \frac{\exp(-x/L_{sr})}{L_{sr}} dx \right] = q(t) \quad (60)$$

$$= q(t) \int_{\frac{N_0}{C_d}}^{\frac{N_0}{C_d - P_p}} (C_d - \frac{N_0}{x}) \frac{\exp(-x/L_{sr})}{L_{sr}} dx + \int_{\frac{N_0}{C_d - P_p}}^{\infty} \frac{P_p \exp(-x/L_{sr})}{L_{sr}} dx = P. \quad (61)$$

The optimal C_d that satisfies the average power constraint can be found by solving (61). Since C_s (LHS of (60)) is a decreasing function of x_0 , we use the bisection method to locate x_0 which makes $C_s = q(t)$.

By the analysis in Cases 2.1 and 2.2, we can see that the maximum value of C_d is $\tilde{C}_d^u$. The interval of x_0 can be decided according to the value of C_s with $C_d = \tilde{C}_d^u$, $x_0 = \frac{1}{\tilde{C}_d^u - P_p}$.

If $C_s(C_d = \tilde{C}_d^u, x_0 = \frac{1}{C_d^u - P_p}) > q(t)$, case 2.2 holds. Otherwise, case 2.1 holds.

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